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ENTRANCE EXAMINATION FOR ADMISSION, MAY 2010.

M.Phil./Ph.D. (MATHEMATICS)

COURSE CODE : 252/118

Register Number :

Signature of the Invigilator
(with date)

COURSE CODE : 252/118

Time : 2 Hours Max : 400 Marks

Instructions to Candidates :

1. Write your Register Number within the box provided on the top of this page and fill in the page 1 of the answer sheet using pen.
2. Do not write your name anywhere in this booklet or answer sheet. Violation of this entails disqualification.
3. Read each question carefully and shade the relevant answer (A) or (B) or (C) or (D) in the relevant box of the ANSWER SHEET using HB pencil.
4. Avoid blind guessing. A wrong answer will fetch you –1 mark and the correct answer will fetch 4 marks.
5. Do not write anything in the question paper. Use the white sheets attached at the end for rough works.
6. Do not open the question paper until the start signal is given.
7. Do not attempt to answer after stop signal is given. Any such attempt will disqualify your candidature.
8. On stop signal, keep the question paper and the answer sheet on your table and wait for the invigilator to collect them.
9. Use of Calculators, Tables, etc. are prohibited.

Notation: $\mathbb{R}$ – Real line, $\mathbb{Q}$ – Set of rationals, $\overline{A}$ – closure of A, A^c – complement of A, sp(A) – span of A

IMPORTANT: Mark the correct statement, unless otherwise specified.

1. Let T_1 and T_2 be two topologies on a non-empty set X. Which one of the following is a topology on X?

   (A) $T_1 \cup T_2$ (B) $T_1 \cap T_2$ (C) $(T_1 \setminus T_2)$ (D) (T_2 / T_1)

2. Every compact Hausdroff space is

   (A) discrete (B) connected
   (C) normal (D) locally compact

3. An operator T on a Hilbert space is self-adjoint if and only if for all x, $< Tx, x >$ is

   (A) $\|x\|^2$ (B) real
   (C) purely imaginary (D) none of these

4. An analytic function $f = u + iv$ with constant modulus is

   (A) $u+v$ (B) $\sqrt{u^2+v^2}$ (C) constant (D) none of these

5. An ideal A of a commutative ring R with unity is maximal if and only if R/A is a

   (A) ring (B) group (C) field (D) none of these

6. The residue of $\dfrac{ze^z}{(z-a)^3}$ at $z = a$ is

   (A) e^a (B) $\frac{1}{2}e^a(a+z)$ (C) a (D) $a+z$

7. The value of the integral $\int_c \cos z \, dz$, where c is $\left|z+\frac{1}{2}\right| = \frac{1}{3}$ is

   (A) $2\pi i$ (B) $-2\pi i$ (C) ∞ (D) 0

8. If $\int_c \frac{dz}{z(z+2)} = 0$ then c can be

(A) $|z|=1$ (B) $|z+2|=1$ (C) $|z-1|=2$ (D) $|z-2|=1$

9. The general solution of $x^4 y''' + 2x^3 y'' - x^2 y' + xy = 1$ is

(A) $(c_1 + c_2 \log x)\, x + \frac{c_3}{x} + \frac{1}{4x} \log x$ (B) $(c_1 + c_2 \log x)\, x + 2 \log x + 4$

(C) $(\frac{c_1}{x} + c_2 \log x)\, x + c_3 x^2 + \log x$ (D) $(c_1 + c_2 x^2 \log x)\, x + c_3 x + \log x$

10. If J_n is a Bessel's function then $(x J_n J_{n+1})$ is equal to

(A) $x J_n^2$ (B) $x J_{n+1}^2$ (C) $x\left(J_n^2 + J_{n+1}^2\right)$ (D) $x\left(J_n^2 - J_{n+1}^2\right)$

11. Which one of the following ideals of the ring $Z[i]$ of Gaussian integers is NOT maximal?

(A) $\langle 1+i \rangle$ (B) $\langle 1-i \rangle$ (C) $\langle 2+i \rangle$ (D) $\langle 3+i \rangle$

12. $Z(G)$ denotes the center of a group G, then the order of the quotient group $G/Z(G)$ cannot be

(A) 4 (B) 6 (C) 15 (D) 25

13. Let $Aut\,(G)$ denote the group of automorphisms of a group G. Which one of the following is NOT a cyclic group?

(A) $Aut\,(\mathbb{Z}_4)$ (B) $Aut\,(\mathbb{Z}_6)$ (C) $Aut\,(\mathbb{Z}_8)$ (D) $Aut\,(\mathbb{Z}_{10})$

14. If $A = \begin{pmatrix} 1 & 0 & 0 \\ i & \frac{-1+i\sqrt{3}}{2} & 0 \\ 0 & 1+2i & \frac{-1-i\sqrt{3}}{2} \end{pmatrix}$, then the trace of A^{102} is

(A) 0 (B) 1 (C) 2 (D) 3

15. Which of the following matrices is NOT diagonalizable?

(A) $\begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$ (B) $\begin{pmatrix} 1 & 0 \\ 3 & 2 \end{pmatrix}$ (C) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ (D) $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

16. The subspace $\mathbb{Q}X[0,1]$ of $\mathbb{R}^2$ (with the usual topology) is

(A) dense in $\mathbb{R}^2$ (B) connected (C) separable (D) compact

17. $\mathbb{Z}_2[x]/\langle x^3+x^2+1\rangle$ is

(A) a field having 8 elements (B) a field having 9 elements

(C) an infinite field (D) not a field

18. The number of elements of a principal ideal domain can be

(A) 15 (B) 25 (C) 35 (D) 36

19. The dimension of the vector space $V=\{A=(a_{ij})_{n\times n}:(a_{ij}=-a_{ij})\}$ over the field $\mathbb{R}$ is

(A) n^2 (B) n^2-1 (C) n^2-n (D) $\frac{n^2}{2}$

20. The minimal polynomial associated with the matrix $\begin{pmatrix} 0 & 0 & 3 \\ 1 & 0 & 2 \\ 0 & 1 & 1 \end{pmatrix}$ is

(A) x^3-x^2-2x-3 (B) x^3-x^2+2x-3

(C) x^3-x^2-3x-3 (D) x^3-x^2+3x-3

21. If $A=\begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$ the sum and product of the eigen values of A are

(A) 12, 32 (B) -12, 32 (C) -12, -32 (D) None of these

22. If $A=\begin{pmatrix}3&0&0\\5&4&0\\3&6&1\end{pmatrix}$ the eigen values of $5A$ and A^5 are respectively

(A) 15,20,5; 15,20,5

(B) 3,4,1; 3,4,1

(C) 15, 20, 5; $3^5, 4^5, 1^5$

(D) 3,5,3; $3^5, 5^5, 3^5$

23. The quadratic form of the matrix $\begin{pmatrix}1&0\\0&1\end{pmatrix}$ is

(A) x^2+y^2

(B) $2xy$

(C) x^2+2xy

(D) $(x+y)^2$

24. If A and B are subsets of a metric space then the correct statement is

(A) $Int\,(A\cup B)\subseteq Int\,(A)\cup Int\,(B)$

(B) $Int\,(A\cup B)=Int\,A\cup Int\,B$

(C) $\overline{A}\cup\overline{B}\subseteq\overline{A\cap B}$

(D) $\overline{A\cap B}\subseteq\overline{A}\cup\overline{B}$

25. In $\mathbb{R}$ with discrete metric, the largest bounded set is

(A) $\mathbb{Q}$

(B) $(0,\infty)$

(C) $\mathbb{R}$

(D) None of these

26. In $\mathbb{R}$ with usual metric an example of a compact set is

(A) $\mathbb{Q}$

(B) $\mathbb{R}-\mathbb{Q}$

(C) $\left\{1,\frac{1}{2},\frac{1}{3},\ldots,\frac{1}{n},\ldots\right\}$

(D) $[0,1]$

27. A complete metric space which is not compact is

(A) $(0,1)$

(B) $[0,1]$

(C) $\mathbb{R}$

(D) $\mathbb{Q}$

28. If $f:\mathbb{R}\to\mathbb{R}$ is continuous then

(A) f is 1-1

(B) f is onto

(C) f is uniformly

(D) none of these

29. Let $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be uniformly continuous functions. Then

(A) $f+g$ is uniformly continuous

(B) $f-g$ is uniformly continuous

(C) fg is uniformly continuous

(D) $2f+3g$ is uniformly continuous

30. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function and let $A = \{x \in \mathbb{R} : f(x) = 0\}$. Then

(A) A is closed

(B) A is open

(C) A is bounded

(D) A is compact

31. Let $f(z) = \cos z = u + iv$, where u and v are the real and imaginary parts of $f(z)$. Then

(A) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$

(B) $\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}$

(C) $\frac{\partial u}{\partial x} = \frac{-\partial v}{\partial y}$

(D) none of these

32. If $f_i : \mathbb{R} \to \mathbb{R}$ are two maps whose Laplace transforms are F_i for $i = 1, 2$ then the Laplace transforms of $G(x) = \int_0^x f_1(t) f_2(x-t)\, dt$, $x \in \mathbb{R}$ is

(A) $F_1 F_2^2$

(B) $F_1 - F_2$

(C) $F_1 F_2$

(D) $F_1^2 F_2$

33. Let $f(z) = \omega$, $z \in D$, be a 1-1 map from a domain D of the z-plane onto a domain G of the ω-plane. Then the inverse of the map f is analytic in G

(A) if f is analytic on D

(B) if f is analytic on D $f'(z) \neq 0$, $\forall\, z \in D$

(C) if f is analytic on D $f'(z)$ is real, $\forall\, z \in D$

(D) only if f is analytic on D with $|f'(z)| = 1$, $\forall\, z \in D$

34. The Taylor series of $f(z) = \frac{1}{(z-2)(z-3)}$ in powers of $(z-i)$ has radius of convergence

(A) $\sqrt{5}$

(B) $\sqrt{10}$

(C) $\sqrt{2}$

(D) $\sqrt{3}$

35. Let G be a group whose order is p^2 where 'p' is a prime

(A) then the center of G has order 1

(B) G is abelian

(C) G must be cyclic

(D) none of these

Let X be a topological space and A be a non-empty subset of X. If $x \in X \setminus A$ is a limit point of A then

36. (A) there is a sequence in A that converges to X
    (B) (A) is true only if X is a metric space
    (C) (A) is true if X is a first countable space
    (D) (A) is true only if X is a second countable space

37. (A) Continuous image of a separable space is separable
    (B) A subspace of a separable space is separable
    (C) Every metric space is separable
    (D) None of these

38. (A) The set of all irrationals is not separable
    (B) An uncountable discrete space is not separable
    (C) $\mathbb{R}^{100}$ is not separable
    (D) The space $c[0, 1]$ of all continuous real valued function on [0, 1] with sup norm, is not separable

39. (A) Every continuous function from (0, 1) into $\mathbb{R}$ has a continuous extension to [0, 1]
    (B) If A and B are closed subsets of a metric space X then there is continuous function $f : X \to [0,1]$ such that $f \equiv 0$ on A and $f \equiv 1$ on B
    (C) If $f : \{0,1\} \to \{0,1\}$ is the identity map then f can be extended to a continuous map from [0, 1] onto {0, 1}
    (D) None of these

40. (A) [0, 1] and (0, 1) are homomorphic
    (B) [0,1] and [0, 1) are homomorphic
    (C) [0, 1] and $\mathbb{R}$ are homomorphic
    (D) (0,1) and $\mathbb{R}$ are homomorphic

41. Mark the **wrong** statement

(A) $f(x)=\frac{x}{1+|x|}$, $x \in \mathbb{R}$ is a homomorphism of $\mathbb{R}$ onto $(-1, 1)$

(B) If $S^2=\{(x, y, z): x^2+y^2+z^2=1\}$ and $p=(0,0,1)$ then $S^2 \setminus \{p\}$ is homomorphic to $\mathbb{R}^2$

(C) $f(x)=\frac{1}{1-x}$, $x \in (0,1)$ is a homomorphism of $(0, 1)$ onto $(1, \infty)$

(D) The map $f(x)=e^{ix}$, $x \in [0, 2\pi)$ is a homomorphism of $[0, 2\pi)$ onto $\{z \in \mathbb{C} : |z|=1\}$

42. Mark the **wrong** statement

(A) $\mathbb{R}$ is locally compact and locally connected

(B) $\mathbb{R}$ is locally compact but not locally connected

(C) $\mathbb{R}$ is not locally compact but is locally connected

(D) $\mathbb{R}$ is neither locally compact nor locally connected

43. If $f(x)=\begin{cases} \sin\frac{1}{x} & if \;\; x \neq 0 \\ 0 & if \;\; x=0 \end{cases}$ $\quad A^{+}=\{x \in [0,1]: f(x)=1\}$ and $A^{-}=\{x \in [0,1]: f(x)=-1\}$

then,

(A) A^{+} and A^{-} are both empty sets

(B) A^{+} and A^{-} are finite sets

(C) $A^{+} \cup A^{-}=[0,1]$

(D) A^{+} and A^{-} are infinite sets

44. Mark the **wrong** statement

(A) Every countable compact, Hausdoff space is metrizable

(B) $(\mathbb{R}, \tau)$ is metrizable where τ is the topology consisting of the empty set and complements of finite sets

(C) A separable topological space is metrizable

(D) Let $I=[0,1]$. Then the product space I^{I}, with product topology is metrizable

45. Let X and Y be normed linear spaces and $T : X \to Y$ is linear. Then

(A) T is open if T is continuous

(B) T is onto if T is open

(C) T is 1-1 if T is open and continuous

(D) None of these

46. If P is the class of all polynomials and f is a real valued continuous map on [0, 1]. Let $\alpha = \inf_{p \in P} \sup_{t \in [0,1]} |f(t) - p(t)|$. Then,

(A) $\alpha \geq \sup_{t \in [0,1]} |f(t)|$

(B) $0 < \alpha < \sup_{t \in [0,1]} |f(t)|$

(C) $\alpha = 0$

(D) $\alpha \geq 1$

47. If $w(z) = z^2$ is the complex potential in the first quadrant then the streamlines are

(A) hyperbolas (B) ellipses (C) parabolas (D) straight lines

48. If the Helmholtz-Hodge decomposition is $\vec{w} = \vec{u} + \nabla p$ then

(A) $\vec{u}$ is irrotational

(B) $\vec{u}$ is divergence free and parallel to the boundary

(C) $\vec{u}$ is divergence free and normal to the boundary

(D) ∇p is parallel to the boundary

49. In plane poiseuille flow the velocity profile is a

(A) hyperbola (B) ellipse (C) parabola (D) straight line

50. If p is the pressure, ρ is the density and $\vec{u}$ is the velocity of fluid flows and $\hat{n}$ is the unit normal to the boundary then the momentum flux per unit area is

(A) $\rho \vec{u}$

(B) $p \hat{n}$

(C) $\rho \vec{u} \, (\vec{u} . \hat{n})$

(D) $p \hat{n} + \rho \vec{u} \, (\vec{u} . \hat{n})$

51. If $\vec{u}$ is the velocity of fluid flow and J is the Jacobian of the fluid flow map then $\frac{\partial J}{\partial t}$ is equal to

(A) $J(\nabla.\vec{u})$ (B) J (C) $(\nabla.\vec{u})$ (D) $J+(\nabla.\vec{u})$

52. Every nontrivial, loopless connected graph, has at least two vertices which are

(A) end-vertices (B) cut-vertices (C) non-cutvertices (D) none of these

53. Let G be a simple graph of order $p \geq 3$ and $\delta(G) \geq p/2$. Then G is

(A) Bipartite (B) Eulerian (C) Hamiltonian (D) None of these

54. The edge-chromatic number of Petersen graph is

(A) 1 (B) 2 (C) 3 (D) 4

55. Let G be a connected (p, q) – plane graph having f faces. Then $(p-q+f)$ equal to

(A) 1 (B) 2 (C) 3 (D) 4

56. Let f be a flow in a network N. Then f is a maximum flow if and only if N contains no

(A) f-saturated path (B) f-unsaturated path

(C) f-incrementing path (D) none of these

57. Let $(L, \leq)$ be a lattice in which $*$ and $\oplus$ denote the operations of meet and join, respectively. For any $a, b \in L$, $a \leq b$ is equivalent to

(A) $a*b=b$ (B) $a*b=a$

(C) $a \oplus b = a*b$ (D) $a \oplus b = a$

58. Any Boolean algebra is isomorphic to one of the following

(A) switching algebra (B) bounded, complemented lattice

(C) complete, complemented lattice (D) powers et Boolean algebra

59. If the Lagrangian is given by $L=\frac{1}{2}m(\dot{r}^2+r^2\dot{\theta}^2)+\frac{m\mu}{r}$ then the Routhian function is given by

(A) $\frac{1}{2}m(\dot{r}^2+r^2\dot{\theta}^2)$ (B) $\frac{1}{2}m(\dot{r}^2+\dot{\theta}^2)$

(C) $\frac{1}{2}m(\dot{r}^2+r^2\dot{\theta}^2)-c\dot{\theta}$ (D) $\frac{1}{2}m(\dot{r}^2+r^2\dot{\theta}^2)+\frac{m\mu}{r}-c\dot{\theta}$

60. The flow associated with the vector field $X(x_1, x_2)=(x_1, x_2-x_2, x_1)$ is

(A) rotation (B) translation (C) scaling (D) galisean

61. For the cylinder $x_1^2+x_2^2=r^2$, $r>0$ in $\mathbb{R}^3$, α is a geodesic if and only $\alpha(t)$ is the form

(A) $(r\cos(at+b), r\sin(at+b), ct+d)$ (B) $(r\cos(at+b), r\sin(at+b), dt^2$

(C) $(r\cos at, r\sin at, e^t)$ (D) $(r\cos at, r\sin at, ct^3)$

62. The curvature of a circle of radius r is

(A) $\frac{1}{r^2}$ (B) $\frac{-1}{r^2}$ (C) r^2 (D) $\frac{1}{r}$

63. $f(z)=\frac{z+1}{z\sin z}$ has

(A) a pole of order 1 at $z=0$ (B) a pole of order 2 at $z=0$

(C) a pole of order 3 at $z=0$ (D) none of these

64. The function $f(z)=\begin{cases}\frac{(\bar{z})^2}{z^2} & if\ \ z\neq 0\\ 0 & if\ \ z=0\end{cases}$

(A) is not continuous, at 0 (B) is analytic at $z=0$

(C) $\lim_{z\to 0}f(z)=1$ (D) none of these

65. If $(x, y) = \begin{cases} 0 & if \quad y \leq 0 \text{ or } y \geq x^2 \\ 1 & if \quad 0 < y < x^2, \end{cases}$ $\alpha = \lim_{(x,y)\to(0,0)} f(x, y)$ along $y = mx$, $0 < m \leq 1$ and $\beta = \lim_{(x,y)\to(0,0)} f(x, y)$ along $y = 0$ then

(A) $\alpha = \beta = 1$ (B) $\alpha = 1$ $\beta = 0$ (C) $\alpha = \beta = 0$ (D) $\alpha = 0$ $\beta = 1$

66. Let (X, d) be a metric space. Consider $X \times X$ with product topology. Then the distance map $d = X \times X \to [0, \infty)$

(A) is continuous only in the first variable

(B) is continuous in both the variable

(C) is continuous only in the second variable

(D) is not continuous

67. The series $\sum_{n=1}^{\infty} e^{-nz}$ converges in the region

(A) $R < |\text{Re } z| \leq 1, R > 0$ (B) $R < |\text{Im } z| \leq 1, R > 0$

(C) $-1 \leq \text{Re } z < R, R < 0$ (D) $-1 \leq \text{Im } z < R, R < 0$

68. If J is the class of all singleton subsets of $\mathbb{R}$ then the smallest ring (of sets) containing J is

(A) $J \cup \{\phi\}$

(B) class of all subsets of $\mathbb{R}$ including empty set

(C) class of all finite subsets of $\mathbb{R}$ including empty set

(D) none of these

69. If $\vec{u}$ is the velocity of fluid flow and J is the Jacobian of the fluid flow map then the one item which is not equivalent to the others is

(A) $\nabla \cdot \vec{u} \equiv 0$ (B) $J \equiv 1$

(C) $J \equiv 0$ (D) flow is incompressible

70. If W is the specific enthalpy, p is the pressure and ρ is the density of a fluid flow then

(A) $dw = \frac{dp}{\rho}$ (B) $dw = dp$ (C) $dw = d\rho$ (D) $dw = d(p\rho)$

71. If D is the deformation tensor of an incompressible fluid flow then the trace of D is equal to

(A) zero sometimes (B) one always

(C) zero always (D) one some times

72. In Couette flow with velocity $(0, \frac{A}{r} + Br)$ the vorticity is given by

(A) (0, 0, 2B) (B) (0, 0, 2A) (C) (0, 2A, 2B) (D) (0, 0, 2)

73. The complex potential for a potent vortex at origin with circulation Γ is equal to

(A) $\Gamma 2\pi i e^z$ (B) $\Gamma 2\pi i \log^z$ (C) $\frac{\Gamma e^z}{2\pi i}$ (D) $\frac{\Gamma \log^z}{2\pi i}$

74. The order of the θ-method for ODE is $p = 2$ for

(A) $\theta = \frac{1}{2}$ (B) $\theta = 0$ (C) $\theta = 1$ (D) $\theta = \frac{2}{3}$

75. The optimal order of a quadrature formula with γ nodes is

(A) $p = 3\gamma$ (B) $p = 2\gamma$ (C) $p = \gamma$ (D) $p = 4\gamma$

76. An s-step Adams-Bashforth method is an explicit method for

(A) for $s = 1$ and only (B) only for $1 \le s \le 6$

(C) any $s \ge 1$ (D) for no values of $s \ge 1$

77. The Chebyshev polynomials are orthogonal polynomials with weight function

(A) $w(t) = \frac{1}{\sqrt{1-t^2}}$ in $(-1, 1)$ (B) $w(t) = 1$ in $(-1, 1)$

(C) $w(t) = \sqrt{1-t^2}$ in $(-1, 1)$ (D) $w(t) = t$ in $(-1, 1)$

78. If $E \subseteq \mathbb{R}$ is a Lebesgue measurable set, $w \in \mathbb{R}$ and m is the Lebesgue measure on $\mathbb{R}$ then

(A) $E+x$ is a Lebesgue Measurable set and $m(E+x)=m(E)$

(B) $E+x$ is a Lebesgue Measurable set and $m(E+x)=m(E)+x$

(C) $E+x$ need not be a measurable set

(D) None of these

79. Mark **wrong** statement

(A) If $f:[a,b]\to\mathbb{R}$ is differentiable and f' is bounded on $[a,b]$ then there exists $k>0$ such that $|f(x)-f(y)|\le k|x-y|\ \forall\ x, y\in[a,b]$

(B) If $f:\mathbb{R}\to\mathbb{R}$ is a Lebesgue measurable map then $f^{-1}(B)$ is a Lebesgue measurable set for each Borel subset B of $\mathbb{R}$

(C) If $f:[a,b]\to\mathbb{R}$ is a monotone increasing function then $\{x\in[a,b]=f'(x)$ exists$\}$ is a non empty set

(D) If $f:\mathbb{R}\to\mathbb{R}$ is a map then $|f|$ is Lebesgue measurable implies f is Lebesgue measurable

80. Mark **wrong** statement.

If $(I_\alpha)_{\alpha\in\wedge}$ is a collection of pairwise disjoint intervals of positive length in $\mathbb{R}$, then

(A) $I_\alpha \cap Q$ is nonempty for each α

(B) $I_\alpha \cap (\mathbb{R}\setminus Q)$ is nonempty for each α

(C) $\wedge$ is almost a countable set

(D) $\wedge$ is an uncountable set

81. Mark **wrong** statement

The map $P:\mathbb{R}^2\to\mathbb{R}$ given by $p(x, y)=x, \forall\ (x, y)\in\mathbb{R}^2$

(A) is linear but not continuous

(B) is a linear, continuous and open map

(C) is a linear continuous map but is not an open map

(D) is a linear, continuous map that is not a closed map

82. Mark **wrong** statement

(A) Product of connected topological spaces is connected

(B) Product of compact topological spaces is compact

(C) Product of regular topological spaces is regular

(D) Product of normal topological spaces is normal

83. Mark the **wrong** statement

(A) Any polynomial $p : \mathbb{R} \to \mathbb{R}$ has closed graph

(B) The function $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = \begin{cases} 1 & if \quad x \geq 0 \\ 0 & if \quad x < 0 \end{cases}$ has closed graph

(C) Let $X = C'[0,1]$ and $Y = C[0,1]$ both with sup norm, where $C'[0,1]$ is the space of all continuously differentiable functions If $T = X \to Y$ is given by $T(f) = f'$, $f \in X$ then T has closed graph

(D) If X and Y are normed linear spaces and $T : X \to Y$ is a bijective, bounded linear operator then $T^{-1} : Y \to X$ has closed graph

84. Mark the **wrong** statement

If for any positive integer n, $e_n = (0, 0, \ldots, 0, 1, 0, 0, \ldots)$, where 1 is at the nth place

(A) $l_\infty = \overline{sp}\,\{e_n : n \geq 1\}$

(B) $c_0 = \overline{sp}\,\{e_n : n \geq 1\}$

(C) $l_1 = \overline{sp}\,\{e_n : n \geq 1\}$

(D) $l_2 = \overline{sp}\,\{e_n : n \geq 1\}$

85. Let $f : [0,1] \to \mathbb{R}$ be a continuous map and $F(x) = \int_0^x f(t)\,dt$, $x \in [0,1]$. Then

(A) F is continuous but not differentiable on (0, 1)

(B) F is differentiable on (0, 1)

(C) F is discontinuous at all points of [0, 1]

(D) F is continuous on [0, 1] except at $x = 0$ and $x = 1$

86. If X is a normed linear space and $T: X \to X$ is a linear map such that $\sup\{\|Tx\| : \|x - x_0\| < 1\} < \infty$ for some $x_0 \in X$. Then

(A) T is continuous but not uniformly continuous on X

(B) T is continuous at all x, except $x = x_0$

(C) $\sup\{\|Tx\| : \|x\| < 1\} = \infty$

(D) T is uniformly continuous on X

87. Let X be a normed linear space, f be a continuous linear functional on X and x_0 be a fixed element of X. If $T: X \to X$ is defined as $T(x) = f(x)x_0$ then

(A) T is linear and continuous

(B) T is linear but not continuous

(C) T is not linear but continuous

(D) T is neither linear nor continuous

88. If H is a Hilbert space, Y is a closed subspace of H with an orthonormal basis $\{u_\alpha : \alpha \in \Lambda\}$ and $x \in H$. Then

(A) $x - \sum_{\alpha \in \wedge} \langle x, u_\alpha \rangle u_\alpha$ is orthogonal to Y

(B) x is orthogonal to $\sum_{\alpha \in \wedge} \langle x, u_\alpha \rangle u_\alpha$

(C) $\sum_{\alpha \in \wedge} \langle x, u_\alpha \rangle u_\alpha$ is orthogonal to Y

(D) None of these

89. If H is a Hilbert space, Y is a closed subspace of H and $x \in H$ is orthogonal to Y then

(A) $\|x\| > \|x - y\|$ for each $y \in H \setminus \{0\}$

(B) there exists $y \in Y \setminus \{0\}$ such that $\|x - y_0\| = \|x\|$

(C) $\sum_{\alpha \in \wedge} \langle x, u_\alpha \rangle u_\alpha \neq 0$ where $(u_\alpha)_{\alpha \in \wedge}$ is an orthonormal basis of Y

(D) $\|x\| < \|x - y\|$ for each $y \in H \setminus \{0\}$

90. If $A = \{x \in Q : \sqrt{2} < x < \sqrt{3}\}$ then

(A) A is not a closed subset of Q

(B) A is a compact subset of Q

(C) A is not a compact subset of Q

(D) A is not a bounded subset of Q

91. Mark the **wrong** statement

(A) All separable Hilberts spaces are isometrically isomorphic to each other

(B) A separable Hilbert space is finite dimensional

(C) The sequence space l_2 is not separable

(D) $L_2[-\pi, \pi]$ is not separable

92. Let $f_n(x) = \begin{cases} 1 & if \ \ x \geq n \\ 0 & if \ \ x < n \end{cases}$, for any positive integer n, then the sequence $f(n)$

(A) converges uniformly to 0 on $\mathbb{R}$

(B) converges pointwise but not uniformly to 0 on $\mathbb{R}$

(C) converges pointwise but not uniformly to 1 on $\mathbb{R}$

(D) converges uniformly to 1 on $\mathbb{R}$

93. If Y is a closed subspace of a Hilbert space H $x \in H$ and $A = \{y \in Y : x - y$ is orthogonal to $Y\}$ then

(A) A can be empty

(B) A must be a singleton set

(C) A is neither empty nor a singleton set

(D) A is an infinite set

94. Let X and Y be normed linear spaces with X finite dimensional. If $T : X \to Y$ is linear and $B_X = \{x \in X : \|x\| \leq 1\}$ then

(A) B_X is not compact and T is not continuous on X

(B) B_X is compact but T is not continuous on X

(C) B_X is not compact but T is continuous on X

(D) B_X is compact and T is continuous on X

95. If f is Lebesgue integrable on [0, 1] then,

(A) f is continuous on [0, 1]

(B) f is bounded on [0, 1]

(C) $m\{x \in [0,1] \mid f(x) |= \infty\} = 0$

(D) $m\{x \in [0,1] = | f(x) | \geq 1\} \leq 1$

96. $\lim_{n\to\infty} \frac{n^3}{3^n}$ is

(A) ∞ (B) 0 (C) 1 (D) 3

97. If $f(x) = |2x-3|$ and $g(x) = 2(x-1)$ for $x \in \mathbb{R}$ then the graph of f and g intersect

(A) no where (B) exactly at one point

(C) exactly at two points (D) at infinite set of points

98. If $f:[a, b] \to \mathbb{R}$ is a bounded map then f is Riemann integrable on $[a, b]$ if and only if

(A) f is continuous on $[a, b]$

(B) f is continuous almost everywhere on $[a, b]$

(C) f is monotone on $[a, b]$

(D) f is bounded variation over $[a, b]$

99. If f and g are continuous maps from $\mathbb{R}$ into $\mathbb{R}$ and D is a non empty subset of $\mathbb{R}$, the map $h:\mathbb{R}\to\mathbb{R}$ given by $h(x) = \begin{cases} f(x) & if \quad x \in D \\ g(x) & if \quad x \in D^c \end{cases}$

(A) is continuous on $\mathbb{R}$

(B) is continuous on $\mathbb{R}$ if D is an open set

(C) is continuous on $\mathbb{R}$ if D is a closed set

(D) none of these

100. If H is a Hilbert space with a countable orthonormal basis (u_n) and (α_n) is a sequence of scalars then the series $\sum_{n=1}^{\infty} \alpha_n u_n$ converges in H if

(A) $(\alpha_n)_{n=1}^{\infty}$ is bounded (B) $(\alpha_n)_{n=1}^{\infty}$ is convergent

(C) $\alpha_n = +\frac{1}{\sqrt{n}} \;\; \forall \; n \geq 1$ (D) $\alpha_n = \frac{1}{n} \;\; \forall \; n \geq 1$