ENTRANCE EXAMINATION FOR ADMISSION, MAY 2011.

M.Phil./Ph.D. (MATHEMATICS)

COURSE CODE : 252/118

Register Number :

Signature of the Invigilator
(with date)

COURSE CODE : 252/118

Time : 2 Hours | Max : 400 Marks

Instructions to Candidates :

1. Write your Register Number within the box provided on the top of this page and fill in the page 1 of the answer sheet using pen.
2. Do not write your name anywhere in this booklet or answer sheet. Violation of this entails disqualification.
3. Read each question carefully and shade the relevant answer (A) or (B) or (C) or (D) in the relevant box of the ANSWER SHEET using HB pencil.
4. Avoid blind guessing. A wrong answer will fetch you –1 mark and the correct answer will fetch 4 marks.
5. Do not write anything in the question paper. Use the white sheets attached at the end for rough works.
6. Do not open the question paper until the start signal is given.
7. Do not attempt to answer after stop signal is given. Any such attempt will disqualify your candidature.
8. On stop signal, keep the question paper and the answer sheet on your table and wait for the invigilator to collect them.
9. Use of Calculators, Tables, etc. are prohibited.

NOTATION :

$\mathbb{R}$ - Real line, Q- rationals, $\mathbb{N}$ - Natural Numbers, $\mathbb{C}$ - Complex Plane, sp(A)-span A, dim A – dimension of A, A^c- complement of A, $C_K(Q)$- space of K- valued continuous functions on Q.

1. Suppose that $f : \mathbb{R} \to \mathbb{R}$ is a differentiable function such that $f(0) = 1$. If the derivative of f is < 0 for every x in $\mathbb{R}$ then f is

(A) bounded below on $\mathbb{R}$ (B) bounded above on $[0, \infty)$

(C) bounded above on $(-\infty, 0]$ (D) bounded on $\mathbb{R}$

2. Let f be a continuous function on $\mathbb{R}$. Define $G(x) = \int_0^{\sin x} f(t)\,dt$. Then

(A) G is continuous but not differentiable (B) G does not exist

(C) G is differentiable (D) G exists but is not continuous

3. The stream function ψ is defined for a flow which is

(A) incompressible and two-dimensional (B) compressible and two-dimensional

(C) incompressible and three-dimensional (D) compressible and three-dimensional

4. Let p be a polynomial of degree 37 with real coefficients. Then

(A) it has at least one real root (B) it has 37 real roots

(C) it need not have real roots (D) it has no root

5. If $a < 0$, *then* $\lim_{x \to \infty} x^a \log x$ *is*

(A) e (B) 0 (C) 1 (D) ∞

6. The stream function $\psi(x, y)$ for the plane couette flow is given by

(A) $\frac{y^2}{2}$ (B) $\frac{y}{2}$ (C) $\tanh y$ (D) $\sec h^2 y$

7. If $f:\mathbb{R}\to\mathbb{R}$ *is a continuous map, and* $f(Q)\subseteq\mathbb{N}$ *then*

(A) $f(\mathbb{R})=\mathbb{N}$

(B) $f(\mathbb{R})$ is a finite set but not necessarily a constant

(C) f is unbounded

(D) f is a constant

8. Let X be the Banach space of all complex $n\times n$ matrices equipped with the norm

$$\|A\|=\max_{\substack{1\le i\le n\\1\le j\le n}}|a_{ij}|.\ \ \textit{If}\ \ f:X\to\mathbb{C}\ \textit{is}\ \text{defined by}\ f(A)=\textit{Trace A then}$$

(A) f is not linear

(B) f is linear but not continuous

(C) f is a bounded linear functional with $\|f\|=1$

(D) f is a bounded linear functional with $\|f\|=n$

9. Suppose $p>1$. If f belongs to L^p ([0, 1]), then

(A) f is continuous

(B) f belongs to L^q ([0, 1]) for $1\le q\le p$

(C) f belongs to L^q ([0, 1]) for $q\ge p$

(D) $f\in L^\infty([0,1])$

10. The residue of $f(z)=\dfrac{\sin z}{z^8}$ *at* $z=0$

(A) $\dfrac{1}{8!}$ (B) $-\dfrac{1}{8!}$ (C) $-\dfrac{1}{9!}$ (D) $\dfrac{1}{9!}$

11. Let Γ denote the boundary of the square whose sides lie along $x=\pm1$ *and* $y=\pm1$ *where* Γ

is described in the positive sense. Then the value of $\displaystyle\int_\Gamma\frac{z^2}{2z+3}dz$ *is*

(A) $\dfrac{\pi i}{4}$ (B) $2\pi i$ (C) 0 (D) $-2\pi i$

12. If X is the class of all polynomials on [0,1] then
   (A) X is complete when given the sup norm
   (B) X is complete when given the $L_1 - norm$
   (C) X can not be normed so on to make it complete
   (D) X is complete if given L_2 norm

13. The velocity field $(u(x, y), 0)$ in the plane poiscuive flow is given by
   (A) $u = 1 - y^2$ (B) $u = y$ (C) $u = \sin y$ (D) $u = \tanh y$

14. The polynomial $x^n - a$ over Z is irreducible if
   (A) n is a prime number (B) a is a prime number
   (C) both n and a are prime numbers (D) $a = 1$

15. All the eigenvalues of the matrix $\begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ lie in the disc
   (A) $|\lambda + 1| \leq 1$ (B) $|\lambda - 1| \leq 1$ (C) $|\lambda + 1| \leq 0$ (D) $|\lambda - 1| \leq 2$

16. Mark the **wrong** statement
   The Cantor set C is
   (A) is uncountable and compact
   (B) contains a nontrivial line segment
   (C) is uncountable and has Lebesgue measure 0
   (D) contains all its limit points

17. (A) Every regular T_1-space with countable base is metrizable
   (B) $\mathbb{R}$ with the topology generated by the half open intervals of the form $[a, b]$ is metrizable
   (C) $\mathbb{R}$ with co finite topology (open sets are compliments of finite sets and empty set) is metrizable
   (D) Every separable topological space with a countable neighbourhood base at each point is metrizable.

18. Mark the **wrong** statement

(A) Continuous image of connected space is connected

(B) Continuous image of a locally connected space is locally connected

(C) Continuous image of a path connected space is path connected.

(D) If $f:[0,1]\to Q$ is continuous, then the image of f is a singleton set. (Q has the usual subspace topology)

19. (A) $\mathbb{R}\setminus\{0\}$ *is connected*

(B) $\mathbb{R}^2\setminus\{(0,0)\}$ *is disconnected*

(C) $\mathbb{R}^3\setminus\{(x,0,0):x\in\mathbb{R}\}$ *is connected*

(D) $l_2\setminus B$ is disconnected where B is the closed unit ball of the sequence space l_2

20. Mark the **wrong** Statement

Let K and F be fields and let K be an extension of F. Let a be in K with $a\neq 0$. If a is algebraic over F, then

(A) $a+b$ is algebraic over F for every b in K

(B) na is algebraic over F for every integer n

(C) a^n is algebraic over F for every integer n

(D) 1/a is algebraic over F

21. Let G be a group of order n. If $n\leq 4$, then

(A) G is non-abelian

(B) G may be abelian or non-abelian

(C) G is abelian but not necessarily cyclic

(D) G is cyclic

22. (A) $\mathbb{R}$ is homeomorphic to $S'=\{(x,y):|x|^2+|y|^2=1\}$

(B) $\mathbb{R}$ is homeomorphic to $(0,1)$

(C) $\mathbb{R}$ is homeomorphic to $\mathbb{R}^2$

(D) $\mathbb{R}$ is homeomorphic to $[0,1)$

23. Let *X and Y be normed linear spaces and* $T:X\to Y$ *be a linear map*. Then

(A) *T is continuous on X if and only if T(B) is a bounded subset of Y for some ball B of positive radius in X*

(B) *If* $S:Y\to X$ is linear and TS is the identity operator on Y then ST is the identity operator on X

(C) T is continuous on X if kernel of T is closed in X

(D) T open does not imply T is onto

24. Which is the smallest number that can be expressed as sum of two cubes in two different ways?

(A) 343 (B) 927 (C) 1729 (D) 2322

25. A subgroup H of a group G is a normal subgroup if and only if

(A) $Gh = hG$ for every h in H
(B) $gH = Hg$ for every g in G
(C) $gh = hg$ for every h in H and g in G
(D) $GH = HG$

26. Define a relation ~ in a group G as follows : $a \sim b$ if there exists an element c in G such that $ca = bc$. Then the relation is

(A) reflexive but not transitive
(B) symmetric but not transitive
(C) transitive but not symmetric
(D) an equivalence relation

27. If $o(G) = p^2$ where p is a prime number then

(A) G is cyclic
(B) G is abelian
(C) G has no proper nontrivial subgroup
(D) none of these

28. (A) $sp\{x \in X : \|x\| < 1\} \neq sp\{x \in X : \|x\| < \in\}, \textit{for any} \ \in \neq 1$

(B) $sp\{x \in X : \|x\| < 1\} = sp\{x \in X : \|x\| < \in\}, \textit{for every} \in > 0$

(C) $sp\{x \in X : \|x\| < 1\} = sp\{x \in X : \|x\| < \in\}, \textit{only if} \ \in \geq 1$

(D) $sp\{x \in X : \|x\| < 1\} \neq sp\{x \in X : \|x\| = 1\}$

29. Let X be a normed linear space

(A) Every subspace of X is the kernal of some linear functional defined on X
(B) If Y is a subspace of X then $\| x + Y \| = \inf\{\| x - y \| : y \in Y\}$ is a norm on the quotient space X / Y
(C) If Y is a subspace of X with nonempty interior then $Y = X$
(D) The closure of a subspace of X need not be a subspace

30. Let G be a group of order $n\ p^k$ and gcd $(n, p) = 1$. Then the number of p-Sylow subgroups of G is a

(A) divisor of p
(B) number relatively prime to n
(C) power of p
(D) divisor of n

31. Mark the **wrong** Statement

Let G be a group of order 28. Then

(A) There exists only one subgroup of order 7

(B) Any subgroup of order 7 is a normal subgroup

(C) G is not simple

(D) G has no subgroup of order 4

32. $\mathbb{Z}_6$ is a direct product of the groups

(A) $\mathbb{Z}_2$ and $\mathbb{Z}_3$

(B) $\mathbb{Z}_4$ and $\mathbb{Z}_2$

(C) $\mathbb{Z}_2$, $\mathbb{Z}_2$ and $\mathbb{Z}_2$

(D) $\mathbb{Z}_3$ and $\mathbb{Z}_3$

33. The number of elements in a minimum generator set of $Q[\sqrt[3]{2}\,]$ over the field Q is

(A) 1 (B) 2 (C) 3 (D) 6

34. Let K be an extension of F. Then

(A) Every element of K is algebraic over F

(B) For any element a in K every element of $F\,(a)$ is algebraic over F

(C) If an element a in K satisfies a polynomial over F then every element of $F\,(a)$ is algebraic over F

(D) Only the elements of F are algebraic over F

35. Mark the **wrong** statement

(A) $\sin z$ *is an unbounded function on* $\mathbb{C}$

(B) $\{z\in \mathbb{C}:\sin z=0\}=\{(n\pi,0):n=0,\pm 1,\pm 2---\}$

(C) $f(z)=\begin{cases}\dfrac{\sin z}{z-\pi}+1 & z\neq \pi\\ 0 & z=\pi\end{cases}$ *has a pole at* $z=\pi$

(D) $\sin z$ *is periodic*

36. If f is analytic on the open unit disc $D=\{z:|z|<1\}$ *and* $f(\frac{1}{n})=0$ *for* $n=2,3...$ *then*

(A) f is a non- zero constant on D

(B) $f\equiv 0$ *on* D

(C) f vanishes only on the real axis

(D) f is unbounded on D

37. Let K be an extension of a field F and a in K

(A) If a is algebraic over F then a is algebraic over K

(B) If a is algebraic over K then a is algebraic over F

(C) If a is algebraic over some extension of K then a is algebraic over F

(D) If a is algebraic over K then a is algebraic over any extension of F

38. Let K be the splitting field of a polynomial of degree n over a field F. Then the degree $[K : F]$ is

(A) At least n (B) At least $n!$ (C) exactly $n!$ (D) At most $n!$

39. Mark the **wrong** statement

(A) There exists a field having 5 elements

(B) There exists a field having 16 elements

(C) There exists a field having 36 elements

(D) There exists a field having 125 elements

40. The dimension of the vector space V of all skew symmetric $n \times n$ matrices with complex entries over the field R is

(A) n^2 (B) $n^2 - 1$ (C) $n^2 - n$ (D) $\frac{n^2}{2}$

41. 41. In the plane Poiseulle flow inside a pipe of radius 'a' the mass flow is proportional to

(A) a^4 (B) a^3 (C) a^2 (D) a

42. Mark the **wrong** statement

Let $F : D \to \mathbb{C}$ *be a map where* $D = \{z \in \mathbb{C} : |z| < 1\}$. Then

(A) If f is a continuous on D and analytic on $D \setminus \{z \in \mathbb{C} : |\operatorname{Im} z = 0\}$, *then* f *is analytic on* D

(B) If f is analytic on D and $f'(z) \neq 0$ *for all* $z \in D$ *then* f *is a conformal mapping on* D

(C) If f is analytic on D and f has no zeros in D and $|f|$ *attains its* minimum *on* D *then* f is the identify map

(D) If f is analytic on D then $|f|$ does not attain its supremum on D.

43. Let $f_n : [0, 1] \to \mathbb{R}$ be a Lebesgue measurable map for each n in N and assume (f_n) converges point wise to a map f on $[0, 1]$. Let $E_n = \bigcup_{k \geq n} \{x \in [0,1] : |f_k(x) - f(x)| > 1\}$, for n in N. Then

(A) E_n is not a Lebesgue measurable set for every n

(B) $(E_n)_{n=1}^{\infty}$ is a decreasing sequence of measurable sets and $\lim_{n \to \infty} m(E_n) = 0$

(C) $(E_n)_{n=1}^{\infty}$ is a decreasing sequence of measurable sets and $\lim_{n \to \infty} m(E_n) = 1$

(D) E_n is a measurable set for each n and $\lim_{n \to \infty} m(E_n) = \infty$

44. Let $C'[0,1]$ denote the space of all real valued, continuously differentiable functions on $[0,1]$ with sup norm and $C[0,1]$, the space of all continuous, real valued functions on $[0, 1]$ with sup norm. Suppose $Tf(x) = f'(x) + \int_0^x f(t)\, dt,\ for\ x \in [0,1]$. Then

(A) T is linear but does not have a closed graph

(B) T is linear, has a closed graph but not continuous

(C) T is linear and continuous

(D) T is not linear

45. Fermat's little theorem states, if p is prime $a \equiv 1(\text{mod } p)$ *then*

(A) $a^{p-1} \equiv 1(mod\ p)$

(B) $a^{p+1} \equiv 1(mod\ p)$

(C) $a^{1-p} \equiv 1(mod\ p)$

(D) $a^{p} \equiv 1(mod\ p)$

46. Mark the **wrong** statement

Let $f : \mathbb{C} \to \mathbb{C}$ *be a map*

(A) If f is an entire function and doubly periodic then, f is a constant

(B) If f is an entire function and f has infinite number of zeroes in the unit disc then f is a constant

(C) If f is an entire function and $\dfrac{f(z)}{z^3}$ is bounded in the region $|z| \geq 1$ then f is a constant

(D) If f is an entire bounded function then f is a constant

47. In Z_{12}, the set of zero divisors is exactly equal to

(A) $\{0\}$ (B) $\{5,7,11\}$

(C) $\{5,7\}$ (D) $\{2,3,4,6,8,10\}$

48. Suppose N is a normal subgroup of a group G such that the factor group $\frac{G}{N}$ is abelian. If G' denotes the commutator subgroup, then

(A) N is a subgroup of G' (B) G' is a subgroup of N

(C) $N = G'$ (D) $N = G$

49. (A) $\frac{\sin z}{z}$ has an isolated removable singularity at $z = 0$

(B) $z = 0$ is an isolated but not removable singularity of $\frac{\sin z}{z}$

(C) $z = 0$ is a removable but not an isolated singularity of $\frac{\sin z}{z}$

(D) For every open set U containing 0, $\sup\{|\frac{\sin z}{z}|: z \in U \setminus \{0\}\} = \infty$

50. If $f(z) = \frac{1}{(z-1)(z-2)}$ then the Laurent series of f centered at $z = 1$ converges in the region

(A) $0 < |z-1| < \frac{5}{2}$ (B) $0 < |z-1| < 1$ (C) $0 < |z-1| < 2$ (D) $0 < |z-1| < \frac{3}{2}$

51. Let G be a group of order 14. Then the number of elements in G of order 7 is

(A) 1 (B) 6 (C) 7 (D) 13

52. In the ring of integers if I = (12) and J = (21) are two principal ideals, then the ideals $I + J$ and $I \cap J$ are respectively given by

(A) (84), (3) (B) (3), (252) (C) (3), (84) (D) None of these

53. Mark **wrong** statement

(A) $\frac{1}{\sin z}$ is a meromorphic function on the complex plane

(B) $e^{1/z}$ is a meromorphic function on the complex plane

(C) Any rational function is meromorphic on the complex plane

(D) Product of two meromorphic functions is again meromorphic.

54. The residue of $f(z) = \dfrac{1}{(z^2+1)^2}$ *at* $z = i$ *is*

(A) i (B) $\dfrac{1}{2i}$ (C) $\dfrac{1}{3i}$ (D) $\dfrac{i}{4}$

55. If $u(x, y) = xy$ is a harmonic function, a harmonic conjucate of u is

(A) $\dfrac{x^2}{2}$ (B) $x^2 + y^2$ (C) $\dfrac{y^2}{2} + \dfrac{x^2}{2}$ (D) $\dfrac{y^2}{2} - \dfrac{x^2}{2}$

56. Let F be a field. When will a polynomial of degree 2 or 3 over F be reducible over F?

(A) always

(B) if and only if it has no root in F

(C) if and only if it has a root in F

(D) if and only if it is monic

57. The number of elements of a principal ideal domain can be

(A) 15 (B) 25 (C) 35 (D) 36

58. The number of non-isomorphic simple graphs on 4 vertices is

(A) 8 (B) 10 (C) 11 (D) 1

59. If m and n are sum of two squares, so is

(A) $m + n$ (B) $m - n$ (C) $n - m$ (D) mn

60. In Couette flow with velocity $(0, \dfrac{A}{r} + Br, 0)$ the vorticity is given by

(a) (0, 0, 0) (B) (0, 0, 2A) (C) (0, 0, 2B) (D) (2A, 2B, 0)

61. If $w(z) = z^2 = (x+iy)^2$ is the complex potential then the streamlines in the first quadrant are given by

(A) $xy = constant$

(B) $x^2 - y^2 = constant$

(C) $x^2 + y^2 = constant$

(D) $x - y = constant$

62. The Legendre polynomials are orthogonal polynomials with weight function $w(t)$ equal to

(A) 1 (B) $\sqrt{1-t^2}$ (C) $\dfrac{1}{\sqrt{1-t^2}}$ (D) $(1-t^2)$

63. The residue at 2 of the function $\dfrac{z+1}{z^2-2z}$ is

(A) $-\dfrac{1}{2}$ (B) 0 (C) 1 (D) $\dfrac{3}{2}$

64. Mark the **Wrong** statement

(A) A tree has a cycle

(B) A tree is connected

(C) A tree is acyclic and connected

(D) If a tree has n vertices, then it has $n-1$ edges

65. The general solution of $\dfrac{dx}{y+z}=\dfrac{dy}{z+x}=\dfrac{dz}{x+y}$ is

(A) $\varphi\left(x+y+z,\ x^2-y^2\right)$

(B) $\varphi\left(\dfrac{y-x}{z-x},\ (y-x)\,(x+y+z)^{\frac{1}{2}}\right)$

(C) $\varphi\left(\dfrac{x-y}{y-z},\ \dfrac{y-z}{z-x}\right)$

(D) $\varphi\left(\dfrac{x+y}{z},\ z\text{-}x-y\right)$

66. Mark the **wrong** statement

Let $f:\mathbb{R}\to\mathbb{R}$ *and* $g:\mathbb{R}\to\mathbb{R}$ be uniformly continuous functions.

(A) $f+g$ is uniformly continuous

(B) $f-g$ is uniformly continuous

(C) fg is uniformly continuous

(D) $2f+3g$ is uniformly continuous

67. If $T:\mathbb{R}^7\to\mathbb{R}^7$ *is linear and* $T^2=0$ *then the rank of* T *is*

(A) ≤ 3 (B) >3 (C) 5 (D) 6

68. Let V be the vector space of 3×3 real matrices $A=(a_{ij})$ s.t $a_{11}+a_{22}+a_{33}=0$. *Then* dim V *is*

(A) 3 (B) 7 (C) 8 (D) 9

69. A nontrivial solution of the equation $u''+q(x)u=0$ *with* $q(x)<0$ *in* $-\infty<x<\infty$ *has*

(A) at most one zero

(B) at least one zero

(C) no zeros

(D) infinitely many zeros

70. Let X have the topology in which the closed sets are the finite sets and the empty set. If (x_n) is a sequence in X such that $x_n \neq x_m$ *if* $n \neq m$ then ,

(A) (x_n) *is not convergent in X*

(B) (x_n) *convergens to a unique element in X*

(C) The set $\{x \in X : x = \lim_{n\to\infty} x_n\}$ is a nonempty finite set but is not a singleton set

(D) The set $\{x \in X : x = \lim_{n\to\infty} x_n\}$ is an infinite set

71. If $\vec{U}$ is the velocity at infinity and Γ is the circulation around the boundary of an olytacle then kutta-joukro ski theorem gives a non zero lift force when

(A) $\vec{U} \neq 0$ *and* $\Gamma \neq 0$

(B) $\vec{U} = 0$ *and* $\Gamma \neq 0$

(C) $\vec{U} \neq 0$ *and* $\Gamma = 0$

(D) $\vec{U} = 0$ *and* $\Gamma = 0$

72. The *bd* $A = \overline{A} \cap \overline{A^c}$. *Then bd* $A = \phi$ if and only if

(A) A is compact

(B) A is finite

(C) A is open and closed

(D) interior of A^c is empty

73. If τ_1 *and* τ_2 *are two topologies on X and* $\tau_1 \leq \tau_2$ *then*

(A) $\tau_2 -$ *closure of a set is contained in its* $\tau_1 -$ *closure*

(B) A sequence that is τ_1*convergent to* $x_0 \in X$ *is* $\tau_2 -$ *convergent to* x_0

(C) The identify map $(X, \tau_1) \to (X, \tau_2)$ *is continuous*

(D) If (X, τ_2) *is regular so is* (X, τ_1)

74. Let (S,d) be a metric space and $B\,(S)$ be the space of all real valued bounded functions defined on S with supnorm. For each

$s \in S$ *define* $f_s(t) = d(s,t)$ *for all* $t \in S$. *Then the map* $s \to f_s$ *from S into* $B(S)$ *is*

(A) $1-1$ but not an isometry

(B) $1-1$ but not continuous

(C) an isometry

(D) continuous but not $1-1$

75. If A is a subset of the real sequence space l_1 given by $A = \{(x_n)_{n=1}^{\infty} \in l_1 : \sum_{n=1}^{\infty} n\, x_n = 0\}$ *then*

(A) The sequence (0,0,1,0,0, ...) is not in the closure of A

(B) The sequence (0,1,0,0, ...) is not in the closure of A.

(C) The sequence (1,0,0...) is in the closure of A

(D) A is closed

76. Let X be a normed linear space X and A be a proper dense subset of the unit sphere $\{x \in X : \|x\| = 1\}$ of X. If diameter of A is sup $\{\|x-y\| : x \text{ and } y \text{ are in } A\}$, then

(A) diameter of A is 2
(B) diameter of $A < 2$
(C) diameter of A is < 1
(D) diameter of A is 1

77. Let (X, d) be a metric space. Then the map $d : X \text{ x } X \to X$ given by $(x, y) \to d(x, y)$, x, y in X is

(A) continuous in both the variables
(B) continuous in the first variable but not in the second variable
(C) continuous in the second variable but not in the first variable
(D) not continuous in both the variables

78. If $A \subseteq X$ *where* X *is a metric space and* $x \in X$ *then*

(A) $d(x, \overline{A}) < d(x, A)$ *and diameter of* $\overline{A} <$ *diameter of* A
(B) $d(x, \overline{A}) > d(x, A)$ *and diameter of* $\overline{A} >$ *diameter of* A
(C) $d(x, \overline{A}) = d(x, A)$ *and diameter of* $\overline{A} =$ *diameter of* A
(D) $d(x, \overline{A}) < d(x, A)$ *and diameter of* $\overline{A} >$ *diameter of* A

79. Let X be a finite set that is not a singleton set. Then the number of Hausdorff topologies that can be defined on X

(A) is finite and more than one
(B) is one
(C) is greater than 1 but less than the candinality of the set X
(D) is the candinality of the power set of X

80. The normal form of the Bessel's equation is $u'' + u = 0$ *when*

(A) $p = 1/2$
(B) $p = 0$
(C) $p = 1$
(D) $p = 4$

81. Let f be an entire function satisfying $|f(z)| \leq k|z|^2$ for some constant k and for all $z \in \mathbb{C}$. Then there is a constant a such that

(A) $f = a$ *on* $\mathbb{C}$
(B) $f(z) = az$ *for all* $z \in \mathbb{C}$
(C) $f(z) = az^2$ *for all* $z \in \mathbb{C}$
(D) $f(z) = e^{az}$ *for all* $z \in \mathbb{C}$

82. If $d(x,y)=|x_1-y_1|+|x_2-y_2|$ *is a metric on* $\mathbb{R}^2$ *where* $x=(x_1,x_2)$ *and* $y=(y_1,y_2)$ *then the set* $S=\{x=(x_1,x_2):d(x,0)=1\}$ is

(A) a circle

(B) an ellipse

(C) a rhombus which is not a rectangle

(D) a rectangle

83. $P(A/B)+P(A/B^c)$ *is*

(A) $P(A)$

(B) 1

(C) Greater than $P(A)$

(D) Less than $P(A)$

84. If X is a finite dimensional normed linear space then

(A) X is homeomorphic to its closed unit ball but not to its open unit ball

(B) X is homeomorphic to its open unit ball and to its closed unit ball

(C) X is homeomorphic to its open unit ball but not to its closed unit ball

(D) X is not homeomorphic to its open unit ball and to its closed unit ball

85. A G_δ set is countable intersection of open sets and F_δ set is countable union of closed sets. Let X be a topological space and $f:X\rightarrow\mathbb{R}$ is a continuous map. Then the set $f^{-1}(\{0\})$

(A) *is a closed*, $G_\delta - set$

(B) is closed but not a $G_\delta - set$

(C) is a compact, $F_\sigma - set$

(D) is compact but not $F_\sigma - set$

86. A solution to the one-dimensional wave equation $\frac{\partial^2 y}{\partial t^2}=\frac{\partial^2 y}{\partial x^2}$ *is*

(A) $\sin(x+t)$

(B) $\sin(x+\pi t)$

(C) $\sin(x-\pi t)$

(D) $\sin(\pi x+t)$

87. Mark the **<u>wrong</u>** statement

Let X be a metric space. Then

(A) Any countable subset is a $G_\delta - set$

(B) Every closed set is a $G_\delta - set$

(C) Every open set is a $F_\sigma - set$

(D) Countable intersection of G_δ *set* is a G_δ *set*

88. Let X be a topological space and A and B be open dense subsets of X. Then $A \cap B$

(A) can be empty

(B) is nonempty open but not necessarily dense in X

(C) is open and dense in X

(D) is dense in $A \cup B$ but not in X

89. If $(f_n)_{n=1}$ is a uniformly bounded sequence of real valued continuous functions on [a,b] and $f = \sup_{1 \le n < \infty} f_n$ *then*

(A) f is uniformly continuous on $[a, b]$

(B) $\{x \in [a,b] : f(x) < \alpha\}$ *is open for each* $\alpha \in \mathbb{R}$

(C) $\{x \in [a,b] : f(x) > \alpha\}$ *is open for each* $\alpha \in \mathbb{R}$

(D) f is continuous but need not be uniformly continuous on $[a, b]$

90. If $\|.\|_1$ *and* $\|.\|_2$ *are two norms on a vector space* X *then the identify map* $i : (X, \|.\|_1) \to (X, \|.\|_2)$ *is continuous if and only if*

(A) 0 is an interior point in the $\|.\|_2$ topology, of the open unit ball of $(X, \|.\|_1)$

(B) 0 is an interior point in the $\|.\|_1$ topology, of the open unit ball of $(X, \|.\|_2)$

(C) The $\|.\|_1$ topology is weaker than the $\|.\|_2$ topology

(D) the identity map $i^{-1} : (X, \|.\|_2) \to (X, \|.\|_1)$ *is continuous*

91. If $\{x_n : 1 \le n < \infty\}$ is an orthogonal subset of a Hilbert space and

$$\| x_n \| = +\frac{1}{\sqrt{2^n}} \text{ for } 1 \le n < \infty \text{ then the series } \sum_{n=1}^{\infty} x_n$$

(A) does not converge in H

(B) converges in H to an element of norm 1

(C) converges in H to an element of norm $\frac{1}{2}$

(D) converges in H to an element of norm $\frac{1}{\sqrt{2}}$

92. Define $f_n:[-1,1]\to\mathbb{R}$ by $f_n(t)=\begin{cases}0 \text{ if } -1\le t<0\\ nt \text{ if } 0\le t<\frac{1}{n}\\ 1 \text{ if } \frac{1}{n}\le t\le 1\end{cases}$. Then the sequence (f_n) of functions

(A) is not bounded

(B) converges point wise to a continuous function on [–1,1]

(C) is Cauchy in the space C ([-1,1]) with sup norm

(D) is Cauchy in C([-1,1]) with the L_2-norm

93. Mark the **wrong** statement

(A) A Hilbert space with an uncountable orthonormal set is not separable

(B) Every orthonormal set in a separable Hilbert space is at most countable

(C) A Hilbert space with a countable orthonormal subset is separable

(D) If H_1 and H_2 are two separable Hilbert spaces then H_1 and H_2 are isometrically isomorphic

94. Mark the **wrong** statement

Let X be a normal Hausdorff topological space. Then

(A) Every subset of X with subspace topology is normal

(B) Every element of X has a closed neighbourhood base

(C) If $(A_i)_{i=1}^n$ is a finite collection of pair wise disjoint closed subsets of X they can be separated by pair wise disjoint open subsets of X

(D) X is completely regular

95. Mark the **wrong** statement

Let X be a topological space, A, a closed subset of X and $U \supseteq A$ be an open subset of X. Then there is a continuous map $f: X\to[0,1]$ *such that* $f\equiv 0$ *on* A *and* $f\equiv 1$ *on* U^c *if*

(A) X is Hausdorff

(B) X is discrete

(C) X is T_1 and normal

(D) X is metrizable

96. Mark the **wrong** statement

(A) A compact, locally connected space has only finite number of components

(B) Q with usual subspace topology has only finite number of components

(C) The components of a locally connected space are both open and closed

(D) $\mathbb{R}$ is locally connected

97. Let P_n denote the class of all polynomials of degree $\leq n$ (with real coefficients) and let $T = \{t_1..t_m\}$ be a set of m distinct reals. $For\, P\ and\ q\ in\ P_n\ set < p,q >= \sum_{t\in T} p(t)q(t)$.

Then

(A) $<..>$ $is\ an\ inner\ product\ on\ T\ for\ any\ m \in \mathbb{N}$

(B) $<..>$ $is\ an\ inner\ product\ if\ m < n$

(C) $<..>$ $is\ an\ inner\ product\ if\ m = n$

(D) $<..>$ $is\ an\ inner\ product\ if\ m \geq n+1$

98. Let H be a Hilbert space and $C \subseteq H$ be a closed convex set. If $x \in H\ the\ set\ x + C$

(A) has a unique element of smallest norm

(B) has a unique element of smallest norm only if $x = 0$

(C) has an element of smallest norm but that need not be unique

(D) has no element of smallest norm

99. Let H be an Hilbert space, Y be a closed subspace of H and

$$x \in H. Then\ y_0 \in Y\ satisfies \| x - y_0 \| = \inf_{y \in Y} \| x - y \|$$

if and only if

(A) $< x - y_0, y > = 0\ \forall y \in Y$

(B) $< x - y_0, y > \geq 0\ \forall y \in Y$

(C) $< x - y_0, y > \leq 0\ \forall y \in Y$

(D) $< x, y_0 - y > = 0\ \forall y \in Y$

100. The value of $p(4)$ where p is the number of possible partitions of 4 is

(A) 4 (B) 1 (C) 5 (D) 10