ENTRANCE EXAMINATION FOR ADMISSION, MAY 2012.

Ph.D. (Mathematics)

COURSE CODE : 118

Register Number :

Signature of the Invigilator
(with date)

COURSE CODE : 118

Time : 2 Hours | Max : 400 Marks

Instructions to Candidates :

1. Write your Register Number within the box provided on the top of this page and fill in the page 1 of the answer sheet using pen.
2. Do not write your name anywhere in this booklet or answer sheet. Violation of this entails disqualification.
3. Read each of the question carefully and shade the relevant answer (A) or (B) or (C) or (D) in the relevant box of the ANSWER SHEET using HB pencil.
4. Avoid blind guessing. A wrong answer will fetch you –1 mark and the correct answer will fetch 4 marks.
5. Do not write anything in the question paper. Use the white sheets attached at the end for rough works.
6. Do not open the question paper until the start signal is given.
7. Do not attempt to answer after stop signal is given. Any such attempt will disqualify your candidature.
8. On stop signal, keep the question paper and the answer sheet on your table and wait for the invigilator to collect them.
9. Use of Calculators, Tables, etc. are prohibited.

Notation : **R** – Real line, **Q** - Set of rationals, **Q** - Set of natural numbers and **C** - Set of Complex numbers, **Z** - Set of integers

For a complex number z, Re z and Im z denote the real and imaginary parts of z respectively.

If G is a group, o(G) denotes the order of G.

For a set E, $\overline{E}$ – closure of E, E^C – complement of E and sp(E) – span of E , dim E – dimension of E.

For a normed linear space X, X^* denotes its dual space, C(Q) – space of scalar valued continuous functions on Q.

Instructions to candidates :

(i) Answer all questions.

(ii) Each correct answer carries 4 marks and each wrong answer carries –1 mark.

(iii) IMPORTANT : Mark the correct statement, unless otherwise specified.

1. Let D be a subset of **R** and let f be a continuous mapping from D to **R** given by $f(x) = x^2 + 1$. Then f is uniformly continuous if D is

   (A) **R**
   (B) Closed and bounded
   (C) $(0, \infty)$
   (D) Any closed interval

2. If $\{v_1, v_2, \ldots v_n\}$ is a basis of a vector space V and if T is a linear operator defined on V by $T(v_i) = v_{i+2}$ if $1 \le i \le n-2$ and $T(v_i) = 0$ if $i \ge n-1$. Then the rank of T^2 is

   (A) n
   (B) $n-3$
   (C) $n-2$
   (D) $n-4$

3. The closure of (0,1] in **R** when **R** is given discrete topology, is

   (A) (0,1)
   (B) [0,1]
   (C) (0,1]
   (D) **R**

4. Let (X, d) be a metric space. Fix x in X. Then the map : $y \to d(x, y)$ is

   (A) not bounded
   (B) bounded but not continuous
   (C) continuous but not uniformly continuous
   (D) uniformly continuous

5. Let f be a function defined on $\mathbf{R}$ by $f(x) = x^{\operatorname{sgn} x}$ if $x \neq 0$ and $f(0) = 0$ where $\operatorname{sgn} x$ is the sign function. Then f is continuous

(A) nowhere

(B) everywhere

(C) only at 0

(D) everywhere except at 0

6. If $P_n(x)$ is the Legendre polynomial of degree n then $P_2(x)$ is equal to

(A) $\frac{1}{2}(3x^2 - 1)$

(B) $\frac{1}{2}(3x^2 + 1)$

(C) $\frac{1}{3}(2x^3 - 1)$

(D) $\frac{1}{3}(2x^3 + 1)$

7. Let $f(x) = |x|^3$. Then f is

(A) continuous but not differentiable anywhere

(B) differentiable only at $x > 0$

(C) uniformly continuous on the real line

(D) differentiable at all nonzero x

8. The exponential map from $(\mathbf{R}, +)$ to $(\mathbf{R}/\{0\}, .)$ is

(A) continuous but not a group homomorphism

(B) not continuous but a group homomorphism

(C) a continuous group homomorphism

(D) an onto group homomorphism

9. If D is the open unit disc of $\mathbf{C}$ and f from D into $\mathbf{C}$ is a non- zero map such that $f\left(\frac{1}{2^n}\right) = 0$ for every n, then f is

(A) not continuous on D

(B) unbounded on D

(C) bounded on D

(D) not analytic on D

10. If A is an $n \times n$ idempotent matrix, then A is

(A) the identity matrix

(B) the zero matrix

(C) singular

(D) non singular and triangular

11. Let $\sum a_n$ be a convergent series of nonnegative terms. Then

(A) $\liminf_n na_n = 0$

(B) $\limsup_n na_n = 1$

(C) $0 < \liminf_n na_n < 1$

(D) $\limsup_n a_n > 0$

12. If $n \geq 0$ then the integral $\int_0^\infty e^{-x} x^n dx$

(A) is equal to $n!$ (B) is equal to $(n-1)!$

(C) does not converge (D) is 0

13. Suppose that $f: \mathbf{R} \to \mathbf{R}$ is continuous and $|f(x) - f(y)| \geq \frac{3}{4}|x-y|$ for all x, y in $\mathbf{R}$. Then $f(\mathbf{R})$ is

(A) $\mathbf{R}$ (B) 0

(C) an interval not necessarily $\mathbf{R}$ (D) $[0, \infty)$

14. The map $f(z) = \frac{\sin \pi\sqrt{z}}{\pi\sqrt{z}}$

(A) an entire function of order 1 (B) an entire function of order $\frac{1}{2}$

(C) is not an entire function (D) an entire function of order 2

15. The matrix corresponding to the differential operator D on $P_n([0,1])$ is

(A) idempotent (B) singular (C) not singular (D) diagonal

16. If $f: l_\infty \to \mathbf{R}$ is given by $f(\{x_n\}) = x_2$, then the norm of f is

(A) 1 (B) 0 (C) 2 (D) $\frac{1}{2}$

17. Let T_1, T_2 be two topologies on a non-empty set X. Then

(A) $T_1 \cap T_2$ is empty

(B) $T_1 \cap T_2$ is a topology on X

(C) $T_1 \cup T_2$ is a topology on X

(D) If T_1 contains T_2, then T_1 / T_2 is a topology on X

18. Let H be a Hilbert space and Y is a closed subspace of H. If $f \in Y^*$ has norm 1 then $A = \{\bar{f} \in H^* : \bar{f}$ is a Hahn-Banach extension of $f\}$ then

(A) A is a singleton set and if $\{\bar{f}\} = A$ then there exists a unique $y_0 \in Y$ such that $\bar{f}(x) = \langle x, y_0 \rangle \quad \forall x \in H$

(B) A is a singleton set and if $\{\bar{f}\} = A$ then there exists $y_0 \in H/Y$ such that $\bar{f}(x) = \langle x, y_0 \rangle \quad \forall x \in H$

(C) A may contain more than one element.

(D) A can be empty set

19. Mark the **wrong** statement

If $f : \mathbf{R} \to \mathbf{R}$ is a measurable function and $g : \mathbf{R} \to \mathbf{R}$ is a continuous function, then

(A) the composition g o f is a measurable function

(B) the composition f o g is a measurable function

(C) $f + g$ is a measurable functions

(D) fg is measurable function.

20. Let X and Y be Banach spaces and $f_n : X \to Y$ be a continuous map for each n and let (f_n) converge pointwise to f in X. Then f is continuous on X if

(A) f_n is uniformly continuous on X for each n

(B) f_n is linear for each n

(C) X is locally compact

(D) X is separable.

21. If $\bar{u}$ is the velocity of fluid flow and J is the Jacobian of the fluid flow map, then $\frac{\partial J}{\partial t}$ is

(A) J (B) $J(\nabla.\bar{u})$ (C) zero (D) $\nabla.\bar{u}$

22. The complex potential for a potential vortex at z_0 with circulation Γ is

(A) $\frac{\Gamma}{2\pi i z}$ (B) $\frac{\Gamma \log z}{2\pi i}$ (C) $\frac{\Gamma z}{2\pi i}$ (D) $\frac{\Gamma \log^{(z-z_0)}}{2\pi i}$

23. The set $\{\sqrt{2} + x : x \text{ is rational}\}$ is

(A) closed in $\mathbf{R}$

(B) open in $\mathbf{R}$

(C) both open and closed in $\mathbf{R}$

(D) neither open nor closed in $\mathbf{R}$

24. Mark the **wrong** statement

Let X be a finite dimensional normed linear space. Then

(A) there exists a finite subset $\{g_1, g_2, \ldots g_n\}$ of X^* such that $\bigcap_{i=1}^{n} \ker nal$ of $g_i = \{0\}$

(B) Every subspace of X is closed in X

(C) Every closed ball in X is compact

(D) X is isometrically isomorphic to $\mathbf{R}^k$ where $k = \dim X$

25. Let $T : \mathbf{R}^n \to \mathbf{R}^n$ be a linear map. Then T is

(A) continuous but not uniformly continuous

(B) continuous if and only if range of T is a bounded set

(C) continuous if and only if range of T is compact

(D) uniformly continuous

26. The inverse Laplace transform of $\dfrac{1}{s(s+1)(s+2)}$ is

(A) $\dfrac{1}{2} + e^{t} + \dfrac{1}{2}e^{2t}$

(B) $\dfrac{1}{2} + e^{-t} + \dfrac{1}{2}e^{-2t}$

(C) $\dfrac{1}{2} - e^{-t} - \dfrac{1}{2}e^{-2t}$

(D) $\dfrac{1}{2} - e^{t} + \dfrac{1}{2}e^{2t}$

27. The singular solution of $y = xy' + y'^2$ is

(A) $y = \dfrac{1}{4x^2}$

(B) $y = -\dfrac{1}{4x^2}$

(C) $y = \dfrac{1}{4}x^2$

(D) $y = -\dfrac{1}{4}x^2$

28. Let X be a metric space with metric d. Let x, y belong to X. Then (X, d_1) is a metric space if $d_1(x, y)$ is defined as

(A) $1 - d(x, y)$

(B) $1 + d(x, y)$

(C) $\dfrac{1 - d(x, y)}{1 + d(x, y)}$

(D) $\dfrac{d(x, y)}{1 + d(x, y)}$

29. Let $0 < q < 1$. The sequence $n\left[q + \frac{q}{n}\right]^n$

(A) converges to 0

(B) converges to 1

(C) converges to e

(D) diverges to ∞

30. The number of roots of the equation $z^{10} - z^4 - 5 = 0$ which lie in the interior of the unit circle is

(A) 0

(B) 1

(C) 5

(D) 10

31. If $2^n - 1$ is a prime number then $2^{n-1}(2^n - 1)$ is a

(A) odd number

(B) even number

(C) perfect number

(D) prime number

32. If $J_p(x)$ is the Bessel function of order p then the positive zeros of $J_{-1/2}(x)$ are separated by the distance

(A) π

(B) $\frac{\pi}{2}$

(C) 2π

(D) $\frac{2}{\pi}$

33. Let H be a subgroup of G. Then

(A) The center of G is contained in H

(B) The center of G is contained in the normalizer of H

(C) The normalizer of H is contained in the center of U

(D) The normalizer of H is contained in H

34. (A) Any subgroup of a infinite cyclic group is infinite

(B) An infinite cyclic group has infinite number of generators

(C) An infinite cyclic group has exactly two generators

(D) An infinite cyclic group has infinitely many subgroups

35. (A) Any two finite abelian groups of same order are isomorphic

(B) Any two infinite cyclic groups are isomorphic

(C) Any two infinite abelian groups are isomorphic

(D) Any two group of order four are isomorphic

36. Let H be a subgroup of a group G. Then the set of all left cosets of H form a group under the induced binary operator of G

(A) if H is an abelian subgroup of G

(B) if H is a cyclic subgroup of G

(C) if H is a normal subgroup of G

(D) if H is a finite subgroup of G

37. Every finite group is isomorphic to

(A) a subgroup of the additive group of integers

(B) an additive group of integers modulo n

(C) a subgroup of a finite cyclic group

(D) a subgroup of permutation group of finite order

38. $ax^8 + 2bx^2 + 2c$ is irreducible if

(A) 2 does not divide both a and b

(B) 2 divides a but does not divides c

(C) 2 divides both b and c

(D) 2 divides neither a nor c

39. Mark the **wrong** Statement

(A) If d divides order of a group G, then U has a subgroup of order d

(B) If a prime p divides order of a group G, then G has a subgroup of order p

(C) If p^r where p prime and r positive integer, divides order of a group G, then G has a subgroup of order p^r

(D) If d divides order of a group G and G is cyclic, then G has a subgroup of order d

40. The cancellation law with respect to multiplication is true

(A) only in fields

(B) only in finite fields

(C) in commutative rings

(D) in integral domains

41. A commutative ring with no nonzero proper ideal is

(A) a zero ring

(B) a ring containing p elements where p is a prime number

(C) a finite ring

(D) a field

42. Let K be the splitting field of the minimal polynomial of the cubic root of 5 over the field $\mathbf{Q}$. Then the degree of K over $\mathbf{Q}$ is

(A) 1 (B) 3 (C) 6 (D) 5!

43. In $\mathbf{Z}$, the ring of integers,

(A) all the prime ideals are maximal ideals

(B) all the maximal ideals are of the form pZ, where p is a prime number

(C) every ideal is maximal ideal

(D) there exist no maximal ideal

44. Let $R[x]$ denote the polynomial ring with indeterminate x. Then

(A) If R is a field then $R[x]$ is a field

(B) If R is a finite ring then $R[x]$ is a finite ring

(C) If R is a integral domain then $R[x]$ is an integral domain

(D) If R is an Euclidean domain then $R[x]$ is an Euclidean domain

45. Mark the **wrong** statement.

If F is a subfield of K and K is a subfield of a field L.

(A) If an element x in L is algebraic over K then x is also algebraic over F

(B) If K is finite extension of F and L is a finite extension of K then L is a finite extension of F

(C) If K is algebraic extension of F and L is algebraic extension of K then L is algebraic extension of F

(D) If F, K and L are finite fields then the number of elements in L is a power of number of elements in F

46. Let X be a normed linear space, $X_1 = \{x \in X : \|x\| = 1\}$ and Y be a proper closed subspace of X. For x in X, let $d(x,Y) = \inf\{\|x-y\| : y \in Y\}$. Then

(A) $\sup\{d(x,Y) : x \in X_1\} = 1$ (B) $\sup\{d(x,Y) : x \in X_1\} > 1$

(C) $d(x,Y) > 1$ for any $x \in X_1 / Y$ (D) $d(x,Y) < 1$, for any $x \in X_1$

47. The ring of Gaussian integers $\{a+ib : a, b$ belongs to $\mathbf{Z}\}$ under complex addition and multiplication is not a field because

(A) multiplication is not commutative

(B) multiplicative inverses do not exist

(C) multiplicative identity does not exist

(D) multiplication is not distributive with respect to addition

48. Let a, b, p, q be positive constants. The series $\sum_{1}^{\infty} \frac{1}{(a+n)^p.(b+n)^q}$ is convergent

(A) for $p+q \leq 1$

(B) for $p \leq 1$

(C) for $q \geq 1$

(D) for all p and q

49. 2^{1000} is equal to

(A) 3 (mod 17)

(B) 7 (mod 17)

(C) 1 (mod 17)

(D) 0 (mod 17)

50. Let X and Y be Banach Spaces and $T : X \to Y$ is a linear map. Then T is continuous on X if the composition go T is continuous for each $g \in Y^*$, is an immediate corollary to

(A) Hahn Banach Theorem

(B) Principle of Uniform Boundedness

(C) Closed graph Theorem

(D) Riesz Fisher Theorem

51. Let A be an $n \times n$ matrix which is both Hermitian and unitary. Then

(A) $A^2 = 1$

(B) A is real

(C) The eigenvalues of A are 0, 1, −1

(D) The characteristic and minimal polynomials of A are the same.

52. Mark the <u>**wrong**</u> statement

X is a normed linear space and if C is a convex subset of a normed linear space X then

(A) Closure of C is convex

(B) If $x \in \overline{C}$ and $y \in$ interior of C then $\lambda x + (1-\lambda)y$ is in interior of C for $0 \leq \lambda < 1$

(C) If C is compact then convexhull of $C \cup \{x\}$ is compact for each $x \in X$

(D) Every $x \in X$ has a nearest element from the closure of C

53. $\int_0^{2\pi} \frac{d\theta}{5-4\cos\theta}$ is

(A) π (B) $\frac{2\pi}{3}$ (C) $-\frac{\pi}{3}$ (D) 0

54. Let R be a ring of characteristic p. Then

(A) R has p elements
(B) $a^p = 0$ for every $a \in R$
(C) $(ab)^p = a^p b^p$ for all $a, b \in R$
(D) $(a+b)^p = a^p + b^p R$ for all $a, b \in R$

55. The number of conjugate classes in the symmetric group S_5 is

(A) 5 (B) 7 (C) 10 (D) 25

56. Let G be a group of order 28. Let H, K be subgroups of G of order 4 and 7 respectively. Then

(A) HK is not a subgroup of G
(B) HK is a proper subgroup of G
(C) G = HK
(D) None of these

57. (A) Every quotient group of a non-abelian group is non-abelian

(B) If $\frac{G}{N}$ is a cyclic group, then G is cyclic

(C) If $\frac{G}{Z(G)}$ is cyclic, then G is abelian

(D) In the set of subgroups the relation "is a normal subgroup" is transitive

58. (A) The gamma function is an analytic function with negative integers as the only zeros

(B) The gamma function is an analytic function without any zeros

(C) The gamma function is a meromorphic function with negative integers and 0 as the poles

(D) The gamma function is a meromorphic function with non-negative integers as the only poles

59. Mark the **wrong** statement?

(A) $K_{3,3}$ is planar

(B) $K_5 - e$ is planar for any edge e of K_5

(C) K_5 is non-planar

(D) The Petersen graph is non-planar

60. If f and g are differentiable functions from $\mathbf{R}$ into $\mathbf{R}$ such that $f(0) = \frac{1}{g(0)}$ and $h(x) = f(x)g(x)\sin\frac{\pi}{2}x$ then $h'(0)$ is

(A) $\frac{2}{\pi}$ (B) $\frac{4}{\pi}$ (C) $-\frac{2}{\pi}$ (D) 1

61. If p is pressure, p is density and $\overline{u}$ is the velocity of a fluid, then the momentum flux per unit area crossing the boundary ∂w with unit outward normal $\hat{n}$ is given by

(A) $p\hat{n}$ (B) $p\hat{n} + p\vec{u}(\vec{u}, \hat{n})$ (C) $\rho\vec{u}(\vec{u}, \hat{n})$ (D) $\rho\overline{u}$

62. In plane Poiseuille flow, the velocity profile is a

(A) straight line (B) parabola (C) ellipse (D) hyperbola

63. The value of $\int_{-1}^{1} |z| dz$ along the upper hemisphere $\{z \in \mathbf{C} : |z| = 1$ and Im $z \geq 0\}$ of the unit disc is

(A) 0 (B) 2 (C) 1 (D) ∞

64. Let P be the class of all polynomials on $[0,1]$. Then

(A) P with supnorm is complete

(B) P with L_1-norm is complete

(C) P with L_2-norm is complete

(D) There is no norm $\|.\|$ on P such that $(P, \|.\|)$ is complete

65. Let $f : [a,b] \to \mathbf{R}$ be a map. Then if $f(x) = \frac{\sin x}{x}$ and $g(x) = \frac{1}{x^{3/2}}$, for x in $[1,\infty)$ then

(A) Both f and g are not Lebesgue integrable

(B) Both f and g are Lebesgue integrable

(C) f is Lebesgue integrable but g is not

(D) g is Lebesgue integrable but f is not

66. If $D \subseteq \mathbf{R}$ Lebesgue measureable set and f is a Lebesgue integrable function on D then

(A) f is bounded on D

(B) f is continuous on D

(C) the set $\{x \in D : |f(x)| = \infty\}$ has Lebesgue measure 0

(D) f is Riemann integrable on D if D is a compact interval

67. In the discrete metric space M the diameter of $A = \{1, 3, 5, 9\}$ is ————

(A) 0 (B) 1 (C) 8 (D) 9

68. If $([a_n, b_n])_{n=1}^{\infty}$ is a sequence of pairwise disjoint intervas such that

$0 < b_n - a_n < \frac{1}{n^3}$ $\forall n \geq 1$ then $f = \sum_{n=1}^{\infty} n\chi_{[a_n, b_n]}$, where $\chi_A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{otherwise} \end{cases}$,

(A) is Lebesgue integrable on $\mathbf{R}$.

(B) is not Lebesgue measurable on $\mathbf{R}$

(C) is Lebesgue integrable only on compact intervals of $\mathbf{R}$

(D) is Lebesgue measurable but not Lebesgue integrable on $\mathbf{R}$

69. The transformation $w = \frac{z - (1 + i2)}{z - (1 - i2)}$ maps the upper half plane $\operatorname{Im} z > 0$ onto

(A) $\{w : |w| < 1\}$ (B) $\{w : |w| \leq 1\}$ (C) $\{w : |w| < 1\}^c$ (D) $\{w : |w| \leq 1\}^c$

70. If $f(z) = \frac{(z^2 - 4)(z - 1)^4}{(\sin \pi z)^4}$ then for f,

(A) $z = \infty$ is an isolated singularity

(B) $z = 2$ is a pole of order 4

(C) $z = -2$ is a pole of order 4

(D) $z = 0$ is a pole of order 4

71. Let X and Y be Banach spaces and $T : X \to Y$ be a linear map. If B_x and B_y are the open unit balls of X and Y respectively then T is an open map if and only if

(A) $T(B_X) \supseteq B_Y$

(B) there exists $\delta > 0$ such that $T(\delta B_X) \subseteq B_Y$

(C) there exists $M > 0$ such that for each $y \in Y$ there is an element $x \in X$ with $Tx = y$ and $\|x\| \leq M\|y\|$

(D) there exists $M > 0$ such that for each $y \in Y$, $\|x\| \leq M\|y\|$ for every $x \in X$ that satisfies $Tx = y$

72. Let X be a normed linear space, Y and Z be closed subspaces of X. Then

(A) $Y + Z$ is a closed subspace of X

(B) $Y + Z$ is a closed subspace of X if Y and Z are complete

(C) $Y + Z$ is a closed subspace if the closed unit ball of Y is compact

(D) $Y + Z$ is a closed subspace if X is complete

73. Mark the **wrong** Statement

If X is an infinite dimensional Banach space

(A) X cannot be countable union of compact sets

(B) X has a closed bounded and convex set that is not compact

(C) X has a discontinuous linear functional defined on it

(D) X has a conyex subset that is not connected

74. If Q is a compact Hausdorff space, $q_0 \in Q$ and $\delta_{q0}(f) = f(q_0)$ $\forall$ $f \in C(Q)$ with subnorm, then δ_{q0}

(A) a bounded linear functional with norm strictly greater than 1

(B) a bounded linear functional with norm 1

(C) is a linear functional that is not continuous

(D) The kernel of δ_{q0} is not closed in $C(Q)$

75. If X is an inner product space, x, y are in X with $|\langle x, y\rangle| = \|x\| \, \|y\|$ then

(A) $x = y$ but x need not be 0

(B) x and y are orthogonal

(C) $x = y = 0$

(D) y is in the span of $\{x\}$

76. If H is a Hilbert space and $f \in X^*$ then $\{x \in H : \|x\| = 1$ and $f(x) = \|f\|\}$

(A) can be empty

(B) is a singleton set

(C) is a finite set with more than one element

(D) is a countably infinite set

77. In a discrete topological space the only compact subsets are

(A) finite subsets

(B) singleton sets

(C) infinite sets

(D) the whole set

78. Mark the **wrong** statement

(A) Arbitrary product of compact space is compact

(B) Arbitrary product of connected space is connected

(C) Arbitrary product of sequentially compact space is sequentially compact

(D) Arbitrary product of completely regular space is completely regular

79. Consider **R** with the co-countable topology τ (open sets are complements of countable sets and the empty set). Then the class of all compact subset of $(\mathbf{R}, \tau)$ is

(A) class of all countable subsets of **R**

(B) class of all finite subsets of **R**

(C) class of all subsets of **R**

(D) class of all singleton subset of **R**

80. Mark the **wrong** statement

(A) $\overline{A}$ is totally bounded if A is totally bounded

(B) If $e_n = (0,0,\ldots,1^{\text{nth}},0,0,\ldots)$, for all positive integers n, then $(e_n)_{n=1}^{\infty}$ is a totally bounded subset of the sequence space 1_1

(C) A subset of $\mathbf{R}^n$ is totally bounded if and only if it is bounded

(D) A compact subset of a metric space is totally bounded

81. If $f(z) = \dfrac{\operatorname{Im}(z)}{|z|}$ for $z \neq 0$ and $f(0) = 0$ where z belongs to the complex plane, then f is

(A) continuous everywhere

(B) discontinuous only at zero

(C) continuous only at zero

(D) continuous nowhere

82. The set of 2×2 matrices with determinant 1

(A) is a group under addition

(B) is a non-commutative group under multiplication

(C) is a commutative group under multiplication

(D) is a group neither under addition nor under multiplication

83. Mark the **wrong** statement

(A) totally bounded subset of a metric space is separable

(B) A subspace of a separable topological space is separable

(C) A topological space with countable base is separable

(D) The product of countable family of separable spaces is separable

84. A harmonic conjugate of $u(x, y) = x^2 - y^2 + xy$ is

(A) $x^2 - y^2 - xy$ (B) $x^2 + y^2 - xy$

(C) $2xy + \frac{1}{2}(y^2 - x^2)$ (D) $\frac{1}{2xy} + 2(y^2 - x^2)$

85. Mark the **wrong** statement

Consider **R** with the topology τ generated by half open intervals of the form $[a, b)$, a, b in **R** $a < b$. Then

(A) $(\mathbf{R}, \tau)$ is separable (B) $(\mathbf{R}, \tau)$ is regular

(C) $(\mathbf{R}, \tau)$ is disconnected (D) $(\mathbf{R}, \tau)$ is metrizable

86. A complete metric space with no isolated points is

(A) not connected (B) not compact

(C) not countable (D) not locally compact

87. In the ring of integers if $I = (39)$ and $J = (93)$ are two principal ideals, then the ideals $I + J$ and $I \cap J$ are respectively given by

(A) (3627), (3) (B) (3), (3627) (C) (1209), (3) (D) (3), (1209)

88. The linear fractional transformation that maps

$z_1 = 0, z_2 = 1, z_3 = \infty$ *onto* $w_1 = -1, w_2 = -i$ *and* $w_3 = 1$ respectively is

(A) $z - i$ (B) $\frac{z-i}{z+i}$ (C) $\frac{z-1}{z+1}$ (D) e^z

89. A topological space X is such that disjoint compact subsets of X can be separated by disjoint open subsets of X. Then

(A) X is metrizable

(B) X is normal but need not be metrizable

(C) X is regular but need not be normal

(D) X is Hausdroff but need not be regular

90. Using $u = \frac{W}{y}$ in the partial differential equation $xu_x = u + yu_y$, the transformed equation has the solution W=

(A) $f\left(\frac{x}{y}\right)$ (B) f(x+y) (C) f(x-y) (D) f(xy)

91. The general solution of $\frac{dx}{y+z}=\frac{dy}{z+x}=\frac{dz}{x+y}$ is

(A) $\varphi\left(x+y+z,\ x^2-y^2\right)$

(B) $\varphi\left(\frac{y-x}{z-x},\ (y-x)\ (x+y+z)^{\frac{1}{2}}\right)$

(C) $\varphi\left(\frac{x-y}{y-z},\ \frac{y-z}{z-x}\right)$

(D) $\varphi\left(\frac{x+y}{z},\ z\text{-}x-y\right)$

92. The Laplace transform of e^{-t} cos 2t is

(A) $\frac{s+1}{s^2+2s+5}$

(B) $\frac{s-1}{(s+1)^2}$

(C) $\frac{s^2-2s+3}{s^2+2s+5}$

(D) $\frac{s^2-1}{s^2+4s+5}$

93. The partial differential equation of the family of surfaces Z=(x+y)+Axy is

(A) $xp-yq=0$

(B) $xp-yq=x-y$

(C) $xp+yq=0$

(D) $xp+yq=x+y$

94. The set {1, x + 1, x^2+1, x^3+1, x^4+1, x^5+1} in $P_5([0,1])$, the class of all polynomials of degree ≤ 5 defined on [0,1]

(A) is a basis

(B) is linearly independent but does not span $P_5([0,1])$

(C) is not linearly independent but spans $P_5([0,1])$

(D) neither spans $P_5([0,1])$ nor is linearly independent

95. Mark the **wrong** statement.

The infinite product $\prod_{k=1}^{\infty} 1+w_k$, where ($w_k$) is a sequence in **C**,

(A) Converges if and only if the sequence (w_k) converges to zero

(B) Converges if and only if $\sum_{k=1}^{\infty} Log(1+w_k)$ converges

(C) Converges absolutely if and only if $\prod_{k=1}^{\infty} 1+|w_k|$ converges

(D) Converges if and only if $\sum_{k=1}^{\infty} w_k$ converges

96. If $f(z)=\sum_{n=1}^{\infty}\frac{1}{e^{n}n^{z}}$ $z\in\mathbf{C}$, then

(A) f is not well defined on $\mathbf{C}$

(B) f is not differentiable at zero

(C) f is a meromorphic function on $\mathbf{C}$

(D) f is an entire function on $\mathbf{C}$

97. If a_n is a sequence of non-negative reals then $\liminf_n(1-a_n)$ equals

(A) $1-\liminf_n(a_n)$

(B) $1-\limsup_n a_n$

(C) $1-\liminf_n(-a_n)$

(D) $1+\liminf_n a_n$

98. Let Aut(G) denote the group of automorphisms of a group G. Which one of the following is NOT a cyclic group?

(A) $Aut(\mathbf{Z}_4)$

(B) $Aut(\mathbf{Z}_6)$

(C) $Aut(\mathbf{Z}_8)$

(D) $Aut(\mathbf{Z}_{10})$

99. If $f(z)=\tan z, z\in\mathbf{C}$, then

(A) $z=\frac{\pi}{2}$ is an essential singularity of f

(B) $z=\frac{\pi}{2}+m\pi$ is a simple pole of f for any integer m

(C) $z=\frac{\pi}{2}+\frac{\pi}{2}$ is a double pole of f for $m=1$ or -1

(D) $z=\frac{\pi}{2}$ is a double pole of f

100. Let F be a field. Then

(A) F has an infinite number of ideals

(B) F has exactly two ideals

(C) F has exactly one ideal

(D) F has no ideal
