ENTRANCE EXAMINATION FOR ADMISSION, MAY 2013.

Ph.D. (Mathematics)

COURSE CODE : 118

Register Number :

Signature of the Invigilator (with date)

COURSE CODE : 118

Time : 2 Hours

Max : 400 Marks

Instructions to Candidates :

1. Write your Register Number within the box provided on the top of this page and fill in the page 1 of the answer sheet using pen.
2. Do not write your name anywhere in this booklet or answer sheet. Violation of this entails disqualification.
3. Read each of the question carefully and shade the relevant answer (A) or (B) or (C) or (D) in the relevant box of the ANSWER SHEET using HB pencil.
4. Avoid blind guessing. A wrong answer will fetch you –1 mark and the correct answer will fetch 4 marks.
5. Do not write anything in the question paper. Use the white sheets attached at the end for rough works.
6. Do not open the question paper until the start signal is given.
7. Do not attempt to answer after stop signal is given. Any such attempt will disqualify your candidature.
8. On stop signal, keep the question paper and the answer sheet on your table and wait for the invigilator to collect them.
9. Use of Calculators, Tables, etc. are prohibited.

Notation: R – Real line, Q- Set of rationals, N- Set of natural numbers and C- Set of Complex numbers, Z- Set of integers,

For a set E, $\overline{E}$ - closure of E, E' - complement of E and sp(E)- span of E.

For a normed linear space X, X^* denotes its dual space.

C[0,1] denotes the set of all continuous real function on [0,1]

Instructions to candidates:

(i) Answer all questions.

(ii) Each correct answer carries 4 marks and each wrong answer carries -1 mark.

(iii) IMPORTANT: Mark the correct statement, unless otherwise specified.

1. The characteristic polynomial of $A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ is

(A) x^2 (B) $x^2 - 1$ (C) $1 - x^2$ (D) $x - 1$

2. Let G be a group of order np^r where p does not divide n. Then the number of subgroups of order pr is of the form

(A) 1+k where p does not divide k (B) 1+k where p divides k

(C) k where p does not divide k (D) k where p divides k

3. The eigen values of the matrix I_2 are

(A) 1,–1 (B) –1,–1 (C) –1, 1 (D) 1,1

4. If the eigenvalues of $A = \begin{pmatrix} 3 & 10 & 5 \\ -2 & -3 & 7 \\ 3 & 5 & 7 \end{pmatrix}$ are 2,2,3 then eigen values of A^{-1}andA^2 are

(A) $\frac{1}{2},\frac{1}{2},\frac{1}{3};2^2,2^2,3^2$ (B) $-2,-2,-3;2^2,2^2,3^2$

(C) $\frac{1}{2},\frac{1}{2},\frac{1}{3};-2^2,-2^2,-3^2$ (D) $4,4,9;\frac{1}{2},\frac{1}{2},\frac{1}{3}$

5. If $A = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$ the sum and product of the eigen values of A are

(A) 32,12 (B) –12,32 (C) –12,-32 (D) 12,32

6. In R^2 let $S = (4,0)$ Then $Span(S) =$

(A) S (B) $\{(x,0) | x \in R\}$ (C) $\{(0,y) | y \in R\}$ (D) R^2

7. Consider the Banach space $C[0,\pi]$ with supremum norm. The norm of the linear function $T:[0,\pi]\to R$ given by $T(f)=\int_0^{\pi} f(x)\sin^2 x dx$

(A) 1 (B) π (C) $\pi/2$ (D) 2π

8. Let V be the set of all polynomials of degree $\leq n$ in $R[x]$. Then the dimension of the vector space V is

(A) ∞ (B) n (C) $n+1$ (D) $n-1$

9. In R^3 if S is the subspace spanned by $\{(1,1,1)\}$ and T is the subspace spanned by $\{(1,2,1)\}$ then $\dim(S\cap T)$ and $\dim(S+T)$ are respectively

(A) 1;1 (B) 0;1 (C) 0;2 (D) 1;2

10. The minimal polynomial of $\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix}$ is

(A) $x^2(x-3)$ (B) $x(x-3)^2$

(C) $x(x-3)$ (D) None of the above

11. If G is a simple (p, q) – bipartite graph, then which one of the following is true?

(A) $p\leq\frac{q^2}{4}$ (B) $p\geq\frac{q^2}{4}$ (C) $q\leq\frac{p^2}{4}$ (D) $q\geq\frac{p^2}{4}$

12. If $(L,*,\oplus,0,1)$ is a lattice, then for all elements $x,y\in L, x\leq y \Leftrightarrow x*y$ is one of the following:

(A) 0 (B) 1 (C) x (D) y

13. Any Boolean algebra is isomorphic to which one of the following:

(A) Power set Algebra (B) Switching Algebra

(C) Lattice of any tuples of 0 and 1 (D) any cube

14. Let T_i and T_j be any two topologies on a nonempty set – X. Then which one of the following is true?

(A) $(T_i\cup T_j)$ is a discrete topology on X.

(B) $(T_i\cap T_j)$ is a topology on X.

(C) $(T_i\cup T_j)=\phi$.

(D) $(T_i\cap T_j)$ is an indiscrete topologies on X.

15. Every compact Hausdorff space is

(A) Discrete space
(B) Normal space
(C) Connected space
(D) Locally compact space

16. The statement : $P \to (Q \to R)$ is equivalent to which on e of the following ?

(A) $(P \wedge Q) \to R$
(B) $(P \vee Q) \to R$
(C) $R \to (P \vee Q)$
(D) $R \to (P \wedge Q)$

17. A metric space is always

(A) First countable
(B) Second countable
(C) Lindelof
(D) Separable

18. An analytic function $f = u + iv$ with constant modulus is

(A) $u + v$
(B) $\sqrt{u^2 + v^2}$
(C) constant
(D) zero

19. The number of spanning trees of a complete graph K_n for n $\geq$ 1 is

(A) n^{n-1}
(B) n^{n-2}
(C) n^n
(D) $n!$

20. Let G be a connected (p, q) – plane graph having f faces. Then (p – q + f) equal to

(A) 1
(B) 2
(C) 3
(D) 4

21. All solution of the equation $e^{2z} + e^{z+1} + e^z + e = 0$ are

(A) $Z = \sin((2k+1)\pi i), k \in Z$
(B) $z = \frac{\pi i}{2k+1}, k \in Z$
(C) $Z = (2k+1)\pi i,\ 1 + (2k+1)\pi i,\ k \in Z$
(D) none of the above

22. What is the maximum possible height of a binary tree on n vertices?

(A) $\frac{(n+1)}{2}$
(B) $\frac{(n-1)}{2}$
(C) $\frac{n(n-1)}{2}$
(D) $\frac{n(n+1)}{2}$

23. Let X be a metric space and $A \subseteq X$ be a connected set with at least two distinct points. Then the number of distinct points in A is

(A) Only two
(B) more than two, but finite
(C) countably infinite
(D) uncountable

24. Let X be any topological space and let $A \subseteq X$. Then the interior of A is

(A) $\overline{(A')}$ (B) $\left(\overline{(A')}\right)'$ (C) $\overline{A} \cap \overline{A'}$ (D) $\overline{A} \cup \overline{A'}$

25. If $f: G \to H$ be a group homomorphism then

(A) $f(G)$ is finite only if G is finite
(B) $f(G)$ is infinite only if G is infinite
(C) $f(G)$ is cyclic only if G is cyclic
(D) $f(G)$ is abelian only if G is abelian

26. Which one of the following statements is wrong?

(A) If G is cyclic then every subgroup of G is cyclic.
(B) If every proper subgroup of G is cyclic then G is cyclic
(C) If G is abelian then all subgroups are abelian
(D) If G is infinite cyclic group then every nontrivial subgroup is infinite.

27. Let G be an abelian group.

(A) If $o(G) = n^2$, then G is cyclic
(B) If $o(G) = n^2$ and n is a prime, then G is cyclic
(C) If $o(G) = pq$, where p and q are primes, then G is cyclic
(D) If G is infinite then G is cyclic.

28. A sequence in a metric space is convergent if

(A) it is a bounded sequence
(B) if it is cauchy sequence
(C) the range of the sequence is finite set
(D) if there exists a point p such that every neighborhood of p does not contain a finite number of terms of the sequence.

29. Let p^n divides $o(G)$, p a prime number. Then which one of the following statement is not always true. Then

(A) G has a subgroup of order pr if $r = 1$
(B) G has a subgroup of order pr if $r = n$
(C) G has a subgroup of order pr if $r < n$
(D) G has unique subgroup of order pr if $r = n$

30. A ring with multiplicative identity is a field if

(A) it has no non-zero zero divisors.
(B) very non zero element in the ring has multiplicative inverse
(C) it is integral domain with finite number of elements
(D) every nonzero element of R is a unit.

24. Let X be any topological space and let $A \subsetneq X$. Then the interior of A is

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(D) every nonzero element of R is a unit.

31. Which one of the following statements is true.

(A) Euclidean domains are principal ideal domains.

(B) Principal ideal domains are Euclidean domains.

(C) Unique factorisation domains are Principal ideal domain.

(D) Unique factorisation domains are Euclidean domain.

32. The polynomial $2x^4 + 6x^3 + 15ax + c$ is irreducible polynomial over the field of rational numbers if

(A) a is any integer and $c = 18$

(B) a is any integer and $c = 10$

(C) a is any integer and $c = 15$

(D) a is any integer and $c = 8$

33. A polynomial with coefficients in a ring R, of degree n, has at most n roots if

(A) R is a commutative ring and only if n is a prime.

(B) R is a commutative ring and n any positive integer. n need not be a prime.

(C) R is a field and only if n is a prime.

(D) R is a field and n any positive integer. n need not be a prime.

34. Which one of the following statements is not true.

(A) Any two finite fields having same number of elements are isomorphic.

(B) The number of elements in a finite field is a prime number.

(C) If G is a subgroup of the multiplicative group of all nonzero elements in F, then G is cyclic

(D) The number of elements in a finite field need not be a prime number.

35. If G is a simple graph whose diameter is two then

(A) G is an Eulerian graph

(B) G is a regular graph

(C) $k(G) = k'(G)$ where $k(G)$ and $k'(G)$ denotes the vertex connectivity and edge connectivity of G.

(D) $\chi(G) \leq \Delta(G)$ where $\chi(G)$ and $\Delta(G)$ denotes the chromatic number and maximum degree of G.

36. If G is simple graph whose chromatic number is 10 then

(A) the number of edges in G is at least $\binom{10}{2}$.

(B) Independence number of G cannot be more than 10.

(C) i is non planar

(D) there exist a clique of size 10.

37. For a simple connected graph G, which of the following statement is wrong?

(A) An edge of G is not a cut edge iff it belongs to a cycle.

(B) G is 3-edge connected iff each edge of G is exactly the intersection of two cycles.

(C) If G is critical then no vertex cut is a clique.

(D) If $\alpha(G)$ denotes the independence number of G then, $\frac{n}{\alpha(G)} \leq \chi(G) \leq n - \alpha(G) - 1.$

38. Which of the following statement is true?

(A) The characteristic of any infinite rings is 0.

(B) If R is a field $R(x)$ is a field.

(C) Any Euclidean domian is a P.I.D.

(D) Any U.F.D. is a P.I.D.

39. The range of a continuous real function defined on a connected space is

(A) the real line

(B) an Interval

(C) a closed and bounded set

(D) compact

40. The set of all isolated points of a second countable space is

(A) empty or countable

(B) finite

(C) uncountable

(D) closed and bounded

41. Let X and Y be two non empty sets and let f:X→Y. If Ai⊊X then

(A) $f(\bigcap_i A_i) = \bigcap_i f(A_i)$

(B) $\bigcap_i f(A_i) \subsetneq f(\bigcap_i A_i)$

(C) $\bigcup_i f(A_i) \subset f(\bigcup_i A_i)$

(D) $f(\bigcap_i A_i) \subsetneq \bigcap_i f(A_i)$

42. There exist zero-divisors in

(A) the ring of integers modulo a prime p

(B) the ring of real matrices of order p

(C) the ring of polynomials over a field of characteristics

(D) the ring of entire functions

43. Which of the following two of the following spaces are homeomorphic:

(A) $[0,1]$ & $(0,1)$

(B) $[0,1]$ & $\{0\} \cup \left\{\frac{1}{n} : n \in N\right\}$

(C) $\{0\} \cup \left\{\frac{1}{n} : n \in N\right\}$ & $\{(x, y) \in R^2 : x^2 + y^2 = 1\}$

(D) $(0,1)$ & $\{(x, y) \in R^2 : x^2 + y^2 = 1\} - \{(1,0)\}$

44. On $X = C[0,1]$ define $T: X \to X$ by $T(f)(x) = \int_0^x f(t)dt$, for all f in X. Then

(A) T is 1-1 and onto

(B) T is 1-1 but not onto

(C) T is not 1-1 but onto

(D) T is neither 1-1 nor onto

45. Consider the following subset Y of R^2 given by $Y = \left\{\left(\left(1 - \frac{1}{t}\right)\cos t, \left(1 - \frac{1}{t}\right)\sin t\right)\right\}$.

(A) Y is connected and compact

(B) Y connected but not compact

(C) Y is not connected but compact

(D) Y is neither connected nor compact.

46. If the map $f : R \to R$ is defined as $f(x) = \begin{cases} x + \frac{3}{2} & if\ x \leq \frac{1}{2} \\ \frac{1}{x} & if\ x > \frac{1}{2} \end{cases}$, then

(A) f is a closed map but not continuous

(B) f is neither a closed map nor continuous

(C) f is both closed and continuous

(D) f is continuous but not a closed map.

47. Consider the following two statements:

(i) The intersection of any collection of compact sets of a space is compact.

(ii) Every compact metric space is complete.

Then,

(A) Both (i) and (ii) are true.

(B) (i) is true but (ii) false

(C) (i) is false but (ii) is true

(D) Both (i) and (ii) are false.

48. Which of the following statement is false?

(A) Every subset of Q is Lebesgue measurable

(B) Every subset of $R\backslash Q$ is Lebesgue measurable

(C) Every subset of the Cantor set is Lebesgue measurable.

(D) Every subset of $\{a + b\sqrt{2} \mid a, b \in Q\}$ is Lebesgue measurable

49. Consider the following set of matrices $S = \left\{\begin{pmatrix} x & x \\ x & x \end{pmatrix} \in R\right\}$ with usual matrix addition and matrix multiplication. Then

(A) S is a commutative ring but not an integral domain.

(B) S is an integral domain but not a skew field.

(C) S is a skew field but not a field.

(D) S is a field.

50. The number of groups of order n (up to isomorphism) is

(A) finite for all n

(B) finite only for finitely many values of n

(C) finite for infinitely many values of n, but not for all values of n.

(D) infinite for some value of n.

51. Suppose $X = (1, \infty)$ and $f : X \to X$ is such that $d(f(x), f(y)) < d(x, y)$ for $x \neq y$. Then

(A) f has atmost one fixed point .

(B) f has a unique fixed point, by Banach contraction theorem.

(C) f has infinitely many fixed points

(D) For every $x \in X$,, the sequence $(f^n(x))_{n=1}^{\infty}$ converges to a fixed point.

52. The set of all continuous functions $f : [0,1] \to R$ satisfying $\int_0^1 t^n f(t) dt = 0$ for $n = 1, 2, 3.....$

(A) is empty

(B) is countably infinite

(C) contains a single element

(D) is uncountably infinite

53. If $f(x,y)=\begin{cases}\dfrac{x^2}{x^2+y^2} & if\ (x,y)\neq(0,0)\\ & if\ (x,y)=(0,0)\end{cases}$, then at $(0,0)$

(A) $f_x(0,0)$ and $f_y(0,0)$ do not exist

(B) $f_x(0,0)$ and $f_y(0,0)$ exist and are equal

(C) $\lim_{(x,y)\to(0,0)}$ exists but f is not continuous at $(0,0)$.

(D) f is continuous at $(0,0)$.

54. The value of the integral $\oint_C \dfrac{dz}{(z-1)(z-3)}$, where C the the positively oriented circle $|Z|=3$, is

(A) πi (B) $-\pi i$ (C) $\dfrac{\pi i}{2}$ (D) 0

55. The function $w=e^z$ is

(A) entire and periodic

(B) analytic, periodic & 1-1 mapping

(C) entire, periodic & 1-1 mapping

(D) analytic, periodic & not 1-1 mapping

56. In the interval [-1,1], the series $\sum_{n=1}^{\infty}(-1)^{n-1}\dfrac{x^2+n^2}{n^3}$ is

(A) Uniformly and absolutely convergent

(B) Absolutely convergent but not uniformly convergent

(C) Neither uniformly nor absolutely convergent

(D) Uniformly convergent but not absolutely convergent

57. If $f(z)$ is analytic then f(z) satisfies,

(A) $f_x+if_y=0$ (B) $f_x-if_y=0$ (C) $f_y if_x=0$ (D) $f_y+if_x=0$

58. Radius of convergence of $\sum_0^{\infty}\dfrac{z^n}{2^n+1}$ is

(A) 0 (B) 1 (C) $\dfrac{1}{2}$ (D) 2

59. If $f:G\to C$ is analytic than $f(x)$ is conformal at each point Z_0 af G if

(A) $f(z_0)=0$ (B) $f(z_0)\neq 0$ (C) $f'(z_0)=0$ (D) $f'(z_0)\neq 0$

60. If $f(x) = z^2 \frac{1}{e^z}$, the singularity of f(x) at $z = 0$ is

(A) removable (B) poles (C) essential (D) none

61. Evaluate $\frac{1}{2\pi i} \int \frac{f'(z)}{f(z)}$ $C : |z| = 4$ and $f(z) = \frac{z^2 + 1}{(z^2 + 2z + 2)^2}$

(A) 1 (B) −2 (C) 2 (D) 4

62. Let a and b be two complex numbers. Then $\left|\frac{a-b}{1-\bar{a}b}\right| < 1$ if

(A) $|a| < 1|b| < 1$ (B) $|a| < 1|b| > 1$ (C) $|a| > 1|b| < 1$ (D) $|a| > 1|b| > 1$

63. In cauchy-goursat theorem, we don't need the condition that

(A) $f(x)$continuous (B) $f'(x)$non zero constant

(C) $f'(x)$ continuous (D) $f'(x) \neq 0$

64. How many roots of equation $z^4 - 6z + 3 = 0$ are in $1 < |z| < 2$

(A) 1 (B) 2 (C) 3 (D) 4

65. If is the Jacobian of the fluid flow map and $\vec{u}(\vec{x}, t)$ is the fluid velocity then $\frac{\partial J}{\partial t}$ is equal to

(A) $(\nabla.\vec{u})J$ (B) $(\nabla.(\vec{u}J))$ (C) $\nabla.\vec{u}$ (D) J

66. If $J(\vec{x}, t)$ is the Jacobian of the fluid flow map of an incompressible fluid then

(A) $J(\vec{x}, t) \equiv 1$ (B) $J(\vec{x}, t) \equiv 0$ (C) $J(\vec{x}, t) > 0$ (D) $J(\vec{x}, t) < 0$.

67. The equation of continuity follows from the principle of

(A) Conservation of Momentum (B) Conservation of Mass

(C) Conservation of Energy (D) second law of Thermodynamics

68. In the pipe Poiseuille flow in a pipe of radius 'a' the mass flow rate is proportional to

(A) a^4 (B) a^3 (C) a^2 (D) a

69. If $\vec{U}$ is the constant velocity at infinity and Γ_C is the circulation around the boundary of a body B then Kutta-Joukowski theorem gives a non zero force around a two-dimensional obstacle B when

(A) $\Gamma_C \neq 0$ and $\vec{U} \neq 0$

(B) $\Gamma_C \neq 0$ but $\vec{U} \neq 0$

(C) $\Gamma_C = 0$ and $\vec{U} \neq 0$

(D) Γ_C and $\vec{U} \neq 0$ are arbitrary.

70. If the Helmholtz-Hodge Decomposition theorem $\vec{w} = \vec{u} + \nabla p$, in a domain D, $\vec{u}$ satisfies

(A) $\nabla.\vec{u} = 0$ in D and $\vec{u}.\vec{n} = 0$ on ∂D

(B) $\nabla \times \vec{u} = 0$ in D and $\vec{u}.\vec{n} = 0$ on ∂D

(C) $\nabla.\vec{u} = 0$ in D and $\vec{u}.\vec{n} \neq 0$ on ∂D

(D) $\nabla.\vec{u} \neq 0$ in D and $\vec{u}.\vec{n} = 0$ on ∂D

71. Let $C[0,1]$ be the set of all continuous functions defined on $[0,1]$ on this set, define addition as multiplication point wise. Then $C[0,1]$ is

(A) a group but not a ring

(B) a ring but not an integral domain

(C) a field

(D) an integral domain but not a field

72. In Plane Poiseuille flow between two parallel plates the velocity profile is

(A) a straight line (B) a hyperbola (C) an ellipse (D) a parabola

73. If $\vec{u}$ is the velocity and $\vec{\omega}$ is the vorticity per unit mass of an ideal fluid and external forces are absent the then $\frac{D\vec{\omega}}{Dt}$ is equal to

(A) $(\vec{\omega}.\nabla)\vec{u}$ (B) $(\vec{u}.\nabla)\vec{\omega}$ (C) $\vec{\omega}(\nabla.\vec{u})$ (D) $\vec{u}(\nabla.\vec{\omega})$

74. An eigen function of $y''+\lambda y = 0$, $y(0)=0$ and $y(\pi)=0$ is given by

(A) $\sin\left(\frac{5x}{2}\right)$ (B) $\sin\left(\frac{x}{2}\right)$ (C) $\sin(3x)$ (D) $\sin x$

75. Which one of the following equations is a self-adjoint equation

(A) $y''+xy=0$

(B) $y''-2xy'+y=0$

(C) $(1-x^2)y''-xy'+y=0$

(D) $x^2y''+xy'+(x^2-1)y=0$

76. If $P_3(x)$ is the Legendre Polynomial of degree 3 then $\int_{-1}^{1} P_3^2(x)dx$ is equal to

(A) $\frac{3}{2}$ (B) $\frac{7}{2}$ (C) $\frac{2}{7}$ (D) $\frac{2}{3}$

77. For the Bessel function $J_{\frac{1}{2}}(x)$ the distance between two successive zeros is equal to

(A) π (B) $\sqrt{\pi}$ (C) $\frac{\pi}{2}$ (D) $\frac{\sqrt{\pi}}{2}$

78. The Gamma function $\Gamma(x)$ is not defined for

(A) Non-positive integers (B) Integers

(C) Positive integers (D) Negative numbers

79. Let G be a finite group. Which of the following is not a divisor of o(G).

(A) Number of elements in a conjugate class

(B) Number of p-sylow subgroups, where p is prime.

(C) Number of distinct right cosets of a subgroup in G

(D) None of the above

80. The field extension $Q(\sqrt{n})$ and $Q(\sqrt{m})$ are field isomorphic

(A) if both m and n are odd primes

(B) if m is a prime and m divides n

(C) only if m and n are square free numbers

(D) only if m and n satisfies same irreducible polynomial over Q

81. $Lt_{n\to\infty} \frac{n^2}{2n^2+1} = ?$

(A) 2 (B) ½ (C) 0 (D) 3

82. If the series $\sum_{k=1}^{\infty} a_k$ is convergent then $Lt_{k\to\infty} ca_k = ?$

(A) c^2 (B) ∞ (C) 0 (D) c

83. The series $\sum_{k=1}^{\infty} \frac{1}{K^p}$ is convergent if

(A) $p < 1$ (B) $p \leq 1$ (C) $p > 1$ (D) $p \geq 1$

84. Which of the following function f: $(0,1) \rightarrow R$ is not of bounded variation.

(A) $f(x) = e^x + x^2$ (B) $f(x) = \sin\frac{1}{x}$ (C) $f(x) = \sqrt{x}$ (D) $f(x) = e^x - x^2$

85. Let (X, d) be a topological space. A sequence $\{P_n\}$ in X has a unique limit

(A) If X is first countable (B) If X is compact

(C) If X is second countable (D) If X is Hausdorff

86. Let a be an element in an extension of a field F and $F(a)$ is an n-dimensional vector space over F. If a polynomial $f(x) \in F[x]$ is satisfied by 'a', then the degree of $f(x)$ is

(A) at least n (B) at most n

(C) exactly n (D) is a divisor of n

87. Upto isomorphism, the number of abelian groups of order 65 is

(A) 2 (B) 5 (C) 7 (D) 49

88. Let f:$[0,\infty) \rightarrow$R defined by $f(x) = \begin{cases} \frac{1}{\sqrt{x}} & if\ x \neq 0 \\ 0 & if\ x = 0 \end{cases}$. Consider the two improper integrals

$I_1 = \int_0^1 f(x)dx$ and $I_2 = \int_1^\infty f(x)dx$. Then

(A) Both I1 and I2 exist (B) I_1 does not exist and I2 exist

(C) I_1 exist but I2 does not (D) neither I1 nor I2 exist

89. Let $f : [a,b] \rightarrow R$ be a monotonic function, then

(A) f is continuous

(B) f is discontinuous at atmost two points

(C) f is discontinuous at finitely many points

(D) f is discontinuous at atmost countable points

90. In a metric space (X,d)

(A) Every infinite set E has a limit point in E

(B) Every closed subset of a compact set is compact

(C) Every closed and bounded set is compact

(D) Every subset of a compact set is closed

91. If a polynomial $f(x)$ with coefficient in Z/pZ has n distinct roots, then the degree of the polynomial is

(A) Greater than or equal to n

(B) Less than or equal to n

(C) Exactly equal to n

(D) Does not depend upon n

92. In a commutative ring

(A) Every prime ideal is maximal

(B) Every non zero prime ideal is maximal

(C) Every maximal ideal is prime

(D) We may have prime ideal which are not maximal and maximal ideal which are not prime.

93. A Scleronomous constraints have

(A) explicit time dependence

(B) no explicit time dependence

(C) non holonomic

(D) rheonomic

94. A particle falling vertically under the influence of gravity when friction forces obtainable from a dissipation function $\frac{1}{2}kv^2$ are present. The maximum possible velocity for fall from rest is

(A) mgk

(B) $\frac{mg}{k}$

(C) $m\sqrt{\frac{g}{k}}$

(D) $\sqrt[k]{mg}$

95. The solution of differential equation $x^4 + zwx'+_0w^2x = 0$ is critically damped if

(A) $w_0 = 0$ (B) $w = 0$ (C) $w = w_0$ (D) $w < w_0$

96. The transformation $Q = \mu(q+ip)$, $P = \upsilon(q-ip)$ is canonical if

(A) $\mu = 1,\ \upsilon = 1$ (B) $\mu = 1,\ \upsilon = \frac{1}{2}$

(C) $\mu = \frac{1}{2},\ \upsilon = 1$ (D) $\mu = 1,\ \upsilon = -\frac{1}{2i}$

97. The dispersion relation of $u_t = u_{xxx}$ is

(A) $\omega = \sqrt{k}$ (B) $\omega = k^2$ (C) $\omega = k^3$ (D) $\omega = \frac{1}{k}$

98. If the Lagrangian is cyclic in q_j, then

(A) p_i is not conserved

(B) p_j is conserved

(C) q_j appear in the Lagrangian

(D) q_j does not appear in the Lagrangian

99. The origin is a zero of $z^3 \sin z$, of order

(A) 1 (B) 3

(C) 4 (D) 0

100. The value of integral $\oint \frac{\cos z}{z(z^2+14)} dz$ along $|z| = 3$ *is*,

(A) 0 (B) $i\pi/2$

(C) $i\pi/7$ (D) none