ਪੰਜਾਬ ਟੈਕਨੀਕਲ ਯੂਨੀਵਰਸਿਟੀ ਜਲੰਧਰ
PUNJAB TECHNICAL UNIVERSITY JALANDHAR

Max. Marks: 90 **Time: 90 Mins.**

<u>Entrance Test for Enrollment in Ph.D. Programme</u>

Important Instructions

- ***Fill all the information in various columns, in capital letters, with blue/black ball point pen.***
- ***Use of calculators is not allowed.***
- ***All questions are compulsory. No negative marking for wrong answers.***
- ***Each question has only one right answer.***
- ***Questions attempted with two or more options/answers will not be evaluated.***

Stream *(Engg./Arch./Pharm./Mgmt./App.Sci./Life Sci.)* APPLIED SCIENCES

Discipline / Branch MATHEMATICS

Name ....

Father's Name ....

Roll No. .... Date: **19-11-2011**

Signature of Candidate ....

Signature of Invigilator ....

Q.1. If a,b,c are the roots of the equation $x^3 - 2x + 5 = 0$, then what is the value of $(a - b)(a - c) + (b - c)(b - a) + (c - a)(c - b)$?

(A) 6
(B) 5
(C) 4
(D) 2

Q.2. The last digit in 3^{400} is;

(A) 7
(B) 9
(C) 1
(D) 3

Q.3. What is the imaginary part of $(\sin 2\theta + i\cos 2\theta)^4 (\sin\theta - i\cos\theta)^{-3}$?

(A) sin 11θ
(B) cos 11θ
(C) – sin 11θ
(D) – cos 11θ

Q.4. The value of 1 + (1 + 2) + (1 + 2+ 3) + -------- + (1 + 2 + 3 + ----- + n) is

(A) $\frac{n(n+1)(2n+1)}{6}$
(B) $\frac{n(n+1)(n+2)}{6}$
(C) $\frac{n(n+1)(2n-1)}{6}$
(D) $\left[\frac{n(n+1)}{6}\right]^2$

Q.5. The number of real solutions of the equation $x^2 - 3|x| + 2 = 0$ are

(A) 0
(B) 2
(C) 3
(D) 4

Q.6. If the set of intergers with the operation defined by m*n = m+n–1 forms a group. What is the inverse of m?

(A) – m
(B) 2 m
(C) 2 – m
(D) 1 – m

Q.7. If I is an identity transformation of finite dimensional vector space v, then

(A) Nullity > dim v
(B) Nullity < dim v
(C) Nullity = dim v
(D) Nullity ≥ dim v

Q.8. The value of the determinant

$$\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}, \text{ is}$$

(A) $a^3 + b^3 + c^3$
(B) 3abc
(C) $a^3 + b^3 + c^3 - 3abc$
(D) $a^3 + b^3 + c^3 + 3abc$

Q.9. The equations $2x - 3y + 6z = 4$, $5x + 7y - 14z = 1$, $3x + 2y - 4z = 0$ have
(A) Unique solution
(B) Exactly two solutions
(C) Infinitely many solutions
(D) No solution

Q.10. If A = $\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$, then A (adj. A) is
(A) $\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$
(B) $\begin{bmatrix} 0 & 10 \\ 10 & 0 \end{bmatrix}$
(C) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
(D) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

Q.11. The matrix A = $\begin{bmatrix} 5 & 3 \\ 3 & 5 \end{bmatrix}$ has eigen values
(A) 4,6
(B) 5,5
(C) 2,8
(D) 1,9

Q.12. Let a 3x3 matrix A have determinant 5, if B = $4A^2$, then the determinant of B is equal to
(A) 20
(B) 100
(C) 320
(D) 1600

Q.13. If $R^2 \rightarrow R$ is a linear map for which T (1,1) = 3 and T(0,1) = – 2, then T(a,b) is equal to
(A) 5a – 2b
(B) 2a – 5b
(C) 2a + 5b
(D) 5a + 2b

Q.14. If z = x + iy and $\left|\frac{1-iz}{z-i}\right|$=1, then z lies on
(A) x – axis
(B) y – axis
(C) a circle with radius unity
(D) on line y = x

Q.15. The analytic function w(z) = u + iv, where v = $\frac{x+y}{x^2+y^2}$, then w(z) is
(A) $\frac{-1+i}{z} + const.$
(B) $\frac{1+i}{z} + const.$
(C) $\frac{1-i}{z} + const.$
(D) $\frac{-1-i}{z} + const.$

Q.16. In the Laurent series expansion of f(z) = $\frac{1}{z-1} - \frac{1}{z-2}$, valid in the region $|z| > 2$, the coefficient of $\frac{1}{z^2}$ is
(A) 0
(B) – 1
(C) 1
(D) 2

Q.17. The value of integral $\oint_{|z|=2} \frac{\cos z}{z^3}$ dz is
(A) πi
(B) $2\pi i$
(C) $-\pi i$
(D) $-2\pi i$

Q.18. If P,Q,A,B are (1,2,5), (–2,1,3), (4,4,2), (2,1,–4) respectively, then the projection of PQ on AB is
(A) 3
(B) 4
(C) 3.5
(D) 4.5

Q.19. What is the equation of the cone with vertex at origin and passing through the circle $x^2 + y^2 = 4$, z =2?
(A) $x^2 + y^2 + z^2 = 0$
(B) $x^2 + y^2 - z^2 = 0$
(C) $x^2 - y^2 + z^2 = 0$
(D) $-x^2 + y^2 + z^2 = 0$

Q.20. Under which one of the following conditions does the equation $ax^2 + by^2 + cx + cy = 0$, $c \neq 0$, represents a pair of straight lines?
(A) a + b + c = 0
(B) a + c = 0
(C) b + c = 0
(D) a + b = 0

Q.21. The directional derivative of $\phi(x,y,z) = 5x^2y - 5y^2z + 2.5\,xz^2$ at the point P (1,1,1) in the direction of the line x – 1 = 2 – y = 2z is
(A) 35/3
(B) 35
(C) 10
(D) 50/3

Q.22. The value of $\oint_c [(2x - y^3)dx + xydy]$, where c is the region enclosed by $|z| = 1$ and $|z| = 3$ is
(A) 60π
(B) 60
(C) 0
(D) 8π

Q.23. $\vec{A} = 2\hat{i} + 3\hat{j} - 4\hat{k}$ and $\vec{B} = 5\hat{i} + 2\hat{j} + 4\hat{k}$, then the angle between $\vec{A}$ and $\vec{B}$ is
(A) π/4
(B) π/3
(C) π/2
(D) π

Q.24. The value of $\int grad(x + y + z) \cdot d\vec{R}$ from (0,1,–1) to (1,2,0) is
(A) 0
(B) 3
(C) – 1
(D) 1

Q.25. If a force $\vec{F} = 2x^2y\hat{i} + 3xy\hat{j}$ displaces a particle in the xy-plane from (0,0) to (1,4) along the curve $y = 4x^2$, then the work done is equal to
(A) 104
(B) 104/5
(C) 32
(D) 32/5

Q.26. A particle executing simple harmonic motion represented by $x = a\sin\omega t$, takes t_1 time in going from x = 0 to x = 0.5a and the time taken from x = 0.5a to x = a it t_2. The ratio of $t_1 : t_2$ would be
(A) 2:1
(B) 1:1
(C) 1:2
(D) 1:3

Q.27. The resultant of two forces acting at angle of 150^0 is 10N and is perpendicular to one of the forces. The other force is
(A) 20N
(B) $20\sqrt{3}N$
(C) $10\sqrt{3}N$
(D) $\frac{20}{\sqrt{3}}N$

Q.28. A particle starting from rest and moving with constant acceleration covers a distance x in first 2 seconds and distance y in next 2 seconds, then
(A) y = x
(B) y = 2x
(C) y = 3x
(D) y = 4x

Q.29. If a train 110 m long passes a telegraph pole in 3 seconds, then the time taken (in seconds) by it to cross a railway platform 165 m long is
(A) 3
(B) 4
(C) 5
(D) 7.5

Q.30. Let f(2) = 4 and $f'(2)=1$, then limit of $\frac{xf(2)-2f(x)}{x-2}$ as x approaches 2 is given by
(A) – 4
(B) – 2
(C) 2
(D) 4

Q.31. The radius of curvature at the origin for the curve $x^3 + y^3 - 2x^2 + 6y = 0$ is
(A) ½
(B) 1
(C) $^3/_2$
(D) 2

Q.32. If $xy - \log_e y = 1$, satisfies the equation $x(yy_2 + y_1^2) - y_2 + \lambda yy_1 = 0$, then λ is equal to
(A) – 3
(B) 1
(C) 3
(D) – 1

Q.33. If f(x) = $\frac{3x+7}{x+2}$, then the value of $\frac{d^4 f(x)}{dx^4}$ at x = 0 is
(A) 3
(B) 3.5
(C) 0.25
(D) 0.75

Q.34. If the point (–1, 2) be a point of inflexion of the function $f(x) = ax^3 + bx^2$, then the value of a and b are:
(A) a = 1, b = 3
(B) a = –1, b = 3
(C) a = –1, b = –3
(D) a ∈ R, b ∈ R

Q.35. The length of the arc $x = t^3$, $y = 2t^2$ from (0,0) to (1,2) is
(A) $^1/_3$
(B) 1
(C) $^{61}/_{27}$
(D) $^{27}/_{61}$

Q.36. The number of real asymptotes of $x^3 + y^3 + 3xy = 0$ are
(A) 0
(B) 1
(C) 2
(D) 3

Q.37. Let f(x) and g(x) be differentiable for 0≤ x ≤ 2, such that f(0) = 4, f(2)=8, g(0) = 0 and $f'(x) = g'(x)$ for all 0≤ x ≤ 2, then the value of g(2) must be
(A) – 4
(B) – 2
(C) 2
(D) 4

Q.38. If $f'(x) = \frac{x^2}{2} - kx + 1$ and f (0) = 0 and f (3) = 15, then the value of k is equal to
(A) 5/3
(B) 3/5
(C) – 5/3
(D) – 3/5

Q.39. For the curve $y^2 = (x-2)(x-5)^2$, the point (5,0) is a
(A) node
(B) single cusp
(C) double cusp
(D) conjugate point.

Q.40. If f (x) is an even function of x in [– π, π], about which axis, will the graph of f (x) is symmetric
(A) x-axis
(B) y-axis
(C) about origin
(D) Both axis

Q.41. If u = $\log_e \left(\frac{x^4+y^4}{x+y}\right)$, show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}$ is equal to
(A) 1
(B) 2
(C) 3
(D) 5

Q.42. If H = f(y – z, z – x, x – y), then the value of $\frac{\partial H}{\partial x}+\frac{\partial H}{\partial y}+\frac{\partial H}{\partial z}$ is equal to
(A) – 1
(B) 0
(C) x + y + z
(D) xyz

Q.43. The maximum value of $\frac{\log_e x}{x}$ is
(A) 1
(B) e
(C) 2/e
(D) 1/e

Q.44. $\lim_{n \to \infty} \sum_{k=1}^{n} \frac{k}{k^2+n^2}$ is equal to
(A) 0
(B) ½ $\log_e 2$
(C) $\tan^{-1} 2$
(D) ∞

Q.45. The area of the region (in square units) bounded by the curve $x^2 = 4y$, line x = 2 and the x-axis is
(A) 1
(B) 2/3
(C) 4/3
(D) 8/3

Q.46. The value of the integral $\int_0^\infty \frac{x^2}{2^x}dx$ is
(A) 2 $\log_e 2$
(B) 2
(C) $(\log_e 2)^{-2}$
(D) $2(\log_e 2)^{-3}$

Q.47. $\int_0^{\pi/2} \frac{\cos 2x}{\cos x+\sin x}dx$ is equal to
(A) – 1
(B) 0
(C) 1
(D) 2

Q.48. $\int_0^{\pi/2} \sin^4 x \cos^2 x dx$ is equal to
(A) 1/6
(B) 1/32
(C) π/32
(D) π/4

Q.49. $\int_e^1 \int_{e^x}^e \frac{dydx}{\log_e y}$ is equal to
(A) e
(B) e – 1
(C) e + 1
(D) 2e

Q.50. The value of $\oint_c z^4 e^{1/z} dz$, where c is $|z| = 1$, is equal to
(A) πi
(B) $\pi i/12$
(C) $\pi i/60$
(D) $-\pi i/60$

Q.51. The particular integral of $\frac{d^2y}{dx^2}+y = xe^{2x}$ is
(A) $\frac{e^x}{25}(5x-4)$
(B) $\frac{e^{2x}}{25}(4x-5)$
(C) $\frac{e^x}{25}(4x-5)$
(D) $\frac{e^{2x}}{25}(5x-4)$

Q.52. The necessary and sufficient condition for equation M (x,y) dx + N (x,y)dy = 0 to be exact is

(A) $\frac{\partial N}{\partial y} = \frac{\partial M}{\partial x}$

(B) $\frac{\partial N}{\partial y} = -\frac{\partial M}{\partial x}$

(C) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

(D) $\frac{\partial M}{\partial y} = -\frac{\partial N}{\partial x}$

Q.53. An integrating factor of the differential equation sinhy dx + coshy dy = 0 is

(A) e^x

(B) x

(C) y

(D) xy

Q.54. For $\frac{d^2y}{dx^2} + 4y = \tan 2x$, solving by variation of parameters. The value of Wronskian is

(A) 1

(B) 2

(C) 3

(D) 4

Q.55. Elimination of a and b from $z = ae^{bt}\sin bx$ gives the partial differential equation

(A) $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial t^2} = 0$

(B) $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial t^2} = 0$

(C) $\frac{\partial z}{\partial x} + z = 0$

(D) $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial t} = 0$

Q.56. The partial differential equation $y^2\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + 7 = 0$ is

(A) Parabolic

(B) Hyperbolic

(C) Elliptic

(D) Poission's

Q.57. Inverse laplace transform of $\frac{1}{s(s^2+1)}$ is

(A) 1 + sin t

(B) 1 – sin t

(C) 1 + cos t

(D) 1 – cos t

Q.58. Laplace transform of unit step function u (t – a), where a is always positive is

(A) 1

(B) e^{-as}

(C) e^{-as}/s

(D) e^{as}/s

Q.59. $\int_0^\infty te^{-3t}\sin t\, dt$, is

(A) 1/50

(B) 1/25

(C) 3/50

(D) 2/25

Q.60. $J_{1/2}(x)$ is equal to

(A) $\sqrt{\frac{2x}{\pi}}$ sin x

(B) $\sqrt{\frac{2}{\pi x}}$ sin x

(C) $\sqrt{\frac{2x}{\pi}}$ cos x

(D) $\sqrt{\frac{2}{\pi x}}$ cos x

Q.61. The polynomial $2x^2 + x + 3$ in terms of Legendre polynomials is
(A) $1/3[4P_2(x)–3P_1(x) + 11P_0(x)]$
(B) $1/3[4P_2(x)+3P_1(x) – 11P_0(x)]$
(C) $1/3[4P_2(x)+3P_1(x) + 11P_0(x)]$
(D) $1/3[4P_2(x)–3P_1(x) – 11P_0(x)]$

Q.62. If $J_n(x)$ is a Bessel function of first kind, then $\int_0^{\pi}[J_{-2}(x) - J_2(x)]dx$, is equal to
(A) 2
(B) – 2
(C) 0
(D) 1

Q.63. The value of $\int_{-1}^{1} P_2^2(x)dx$, is equal to
(A) 0
(B) 1
(C) 2/3
(D) 2/5

Q.64. Let x be a random variable, denote the number of points rolled with an unbiased die. The expected value of $f(x) = 2x^2 + 1$ is
(A) 94
(B) 94/2
(C) 94/3
(D) 94/4

Q.65. If $P(A \cap B) = 0$, then $P(A / A \cup B)$ is equal to
(A) $\frac{P(A)}{P(B)}$
(B) $\frac{P(A)}{P(B) + P(A \cup B)}$
(C) $\frac{P(A)}{P(A) + P(B)}$
(D) $\frac{P(B)}{P(A) + P(B)}$

Q.66. The median of the numbers 11,10,12,13,9,10 is
(A) 10
(B) 11
(C) 10.5
(D) 11.5

Q.67. In a Poisson distribution if $2P(x = 1) = P(x = 2)$, then the variance is
(A) 0
(B) 1
(C) 2
(D) 4

Q.68. If the correlation coefficient between two variables is zero, then the two regression lines are
(A) Parallel
(B) Perpendicular
(C) Coincident
(D) Inclined at 45^0 to each other

Q.69. The order of convergence in Newton-Raphson method is
(A) 0
(B) 1
(C) 2
(D) 3

Q.70. Simpson's '3/8' rule of integration for evaluation of $\int_a^b f(x)dx$ requires the interval (a,b) to be divided into multiples of
(A) 2
(B) 3
(C) 4
(D) 6

Q.71. For $\frac{dy}{dx} = x + y$, y (0) = 0 using Euler's method and step size of 0.2, the value of y (0.6) is
(A) 0.04
(B) 0.28
(C) 0.128
(D) 0.0128

Q.72. If the mean and variance of a Binomial distribution with parameters (n,p) are 40 and 30 respectively then parameters are
(A) (40,0.75)
(B) (30,0.25)
(C) (160,0.25)
(D) (120,0.50)

Q.73. If x be a normal variate with mean 50 and variance 4, then the modal ordinate is equal to

(A) $\frac{1}{50\sqrt{2\pi}}$

(B) $\frac{1}{4\sqrt{2\pi}}$

(C) $\frac{1}{2\sqrt{2\pi}}$

(D) $\frac{1}{\sqrt{2\pi}}$

Q.74. In testing the significance of the difference of two sample means in case of small samples of size n_1 and n_2, the degree of freedom is calculated by

(A) $n_1 + n_2 - 2$
(B) $n_1 + n_2$
(C) $n_1 + n_2 - 1$
(D) $n_1 - n_2 + 2$

Q.75. Given $x_1+x_2+x_3=4$, $2x_1+x_2+5x_3=5$, the maximum number of possible basic solution is equal to

(A) 3
(B) 4
(C) 2
(D) 6

Q.76. In a balanced transportation problem with m origins and n destinations, the solution is non-degenerate, if the number of occupied cells is equal to

(A) m + n
(B) m + n + 1
(C) m + n – 1
(D) m – n + 1

Q.77. If $u+3x = 5$, $2y - v = 7$ and $r_{xy} = 0.12$ then r_{uv} is equal to

(A) 0.12
(B) – 0.12
(C) 0
(D) Can't be obtained

Q.78. Consider two regression lines $3x + 2y = 26$ and $6x + y = 31$, then the correlation coefficient between two variables and the estimate of y for x = 0 are

(A) 0.5,13
(B) 0.5,31
(C) – 0.5,13
(D) – 0.5,31

Q.79. The number 30.05678 rounded off to four significant figures is

(A) 30.05
(B) 30.0568
(C) 30.06
(D) 30.05670

Q.80. Let $u=5xy^2/z^3$, $\Delta x = \Delta y = \Delta z = 0.001$ and $x = y = z = 1$. Then the relative maximum error is equal to

(A) 0.006
(B) 0
(C) 0.06
(D) 0.03

Q.81. Mean and standard deviation of an examination in which marks 70 and 88 correspond to standard scores of –0.6 and 1.4 respectively are

(A) 79, 9
(B) 79, 3
(C) 75.4, 3
(D) 75.4, 9

Q.82. The number of all possible matrices of order 2 x 2 with each element 0 or 1 is

(A) 2
(B) 8
(C) 16
(D) 32

Q.83. What are the values of a and b respectively, if limit of $\frac{\sin ax - x - \log_e(\cos x)}{x \sin bx}$ as x approaches 0 is ½.

(A) a = 1, b = 1
(B) a = 1, b = ½
(C) a = –1, b = 1
(D) a = –1, b = ½

Q.84. By means of a suitable transform of the independent variable, the differential equation $x\frac{d^2y}{dx^2}+2\frac{dy}{dx}=6x+\frac{1}{x}$ reduces to the form

(A) $\frac{d^2y}{dt^2}+2\frac{dy}{dt}=6e^{2t}+1$

(B) $\frac{d^2y}{dt^2}+\frac{dy}{dt}=6e^{2t}+1$

(C) $\frac{d^2y}{dt^2}=6e^{2t}+\log_e t$

(D) $\frac{d^2y}{dt^2}=6e^{t}+1$

Q.85. The series $1+\frac{2}{6}+\left(\frac{2}{6}\right)\left(\frac{5}{12}\right)+\left(\frac{2}{6}\right)\left(\frac{5}{12}\right)\left(\frac{8}{18}\right)+----\infty$ is

(A) divergent
(B) convergent
(C) oscillates finitely
(D) oscillates infinitely

Q.86. If $\frac{a+b}{a-b}=\frac{1}{5}$, then $\frac{a^2-b^2}{a^2+b^2}$ is equal to

(A) 2:3
(B) 3:2
(C) 5:13
(D) 13:5

Q.87. Two digits are selected at random from the digits (1,2,3,4,5,6,7,8,9) if 2 is the one of the digit selected, what is the probability that sum is odd?

(A) $^5/_8$
(B) ¼
(C) $^2/_5$
(D) $^1/_5$

Q.88. The rational number having the decimal expansion of $0.3\overline{56}$ is

(A) $\frac{48}{495}$

(B) $\frac{353}{990}$

(C) $\frac{3}{10}$

(D) $\frac{35}{99}$

Q.89. If $f(x)=\frac{4^x}{2+4^x}$, then f(x) + f(1 – x) is equal to

(A) 0
(B) 1
(C) 2
(D) 4

Q.90. If $\vec{a},\vec{b},\vec{c}$ be three unit vectors such that $\vec{a}$ x ($\vec{b}$ x $\vec{c}$) = 0.5 $\vec{b}$. What is the angle between $\vec{a}$ and $\vec{c}$?

(A) 30^0
(B) 45^0
(C) 60^0
(D) 90^0