Max. Marks: 90

Time: 90 Mins.

Entrance Test for Enrollment in Ph.D. Programme

Subject *(Engg./Arch./Pharm./Mgmt./Sciences)* **APPLIED SCIENCE**
Discipline / Branch **MATHEMATICS**
Name
Father's Name
Roll No.   Date : **10-07-2010**

Signature of Candidate

Signature of Invigilator

1   Shortest distance between two points in a Riemannian space is

    (a) straight line (b) circle (c) geodesic (d) parabola.

2   Three coins are tossed. What is the probability of getting at least two heads

    (a)1/2   (b)1/8   (c) 3/8   (d) 1

3   How may Mersenne's primes are found till now

    (a) 40   *(b)* 1000

    (c) 47   *(d)*10^5

4   Who have proved the Fermat's Last Theorem

    (a) Fermat   *(b) Andrew While*

    (c) Lagrange   *(d) Euler*

5    $L\{\hat{o}(t)\} = \cdots\cdots\cdots$

    (a) 0   *(b)* 1

    (c)e^{-as} *(d)*$\frac{e^{-as}}{s}$

6    $L^{-1}\left\{\frac{e^{-\pi s/2}}{s^2-1}\right\} = \cdots\cdots\cdots$

    (a) $\sin t$ (b) none

    (c) $-\sin t$ *(d)* $-\cos t$

7   A sample is known as large sample if its size is greater than

    (a) 30   *(b)* 100

    *(c)*1000   *(d)*40

8 One sample $t-test$ is used to compare

(a) means of sample and population   (b) means of two samples

(c) variances of sample and population (d) variances of two samples

9 Perfect fluids are the fluids which have

(a) viscosity   (b) no viscousity

(c) magnetic field   (d) electric field

10 Which one is the normal fuzzy set

(a) {1/.2, 3/.6, 4/1, 6/.9}   (b) {1/.1, 3/.6, 4/.9, 6/.7}

(c) {1/.5, 3/.9, 4/.7, 6/.4}   (d) {1/.6, 3/.7, 4/.5, 6/.3}

11 Intelligence of a person is determined by

(a) size of brain (b) weight of brain (c) neurons in brain (d) location of brain

12 By profession Pierre De Fermat was

(a) mathematician (b) judge (c) carpenter (d) teacher

13 Total number of divisions required to solve an n by n system of equations using Gaussian elimination is

(a) n^2 (b) $(n+1)!$ (c) $n (n+1)/2$ (d) $n(n-1)/2$

14 Total number of additions required to solve an n by n System of equation using Cramer's rule is

(a) n^2 (b) $(n+1)!$ (c) n^3 (d) $n(n-1)/2$

15 Number of edges of graph with n vertices is

(a) $^nC_2$ (b) $^nC_3$ (c) $^nC_1$ (d) $^nC_4$

16 Degrees of freedom is related to:

(a) no. of observations in a set

(b) hypothesis under test

(c) no. of independent observations in a set

(d) none of the above

17 Student's t-test is applicable only when:

(a) the variate values are independent

(b) the variate is distributed normally

(c) the sample is not large

(d) all the above

18 Symbol Σ suggested by

(a) Euler   (b) Bhramagupta

(c) Legendre   (d) Lagrange

19   Unit circle is ....
        (a) Abelian & cyclic Group   (b) Not a Group
        (c) Non Abelian Group   (d) Non cyclic

20   If $x = \xi$ is a root of order m of the equation $f(x) = 0$ then the Newton

        Raphson method will converge

        (a) Linearly   (b) Quadratically   (c) Cubically   (d) Can not be said

21   The Jacobi method for finding eigen values is applicable only if the
        coefficient matrix is
        (a) Hermitian (b) Symmetric (c) Orthogonal (d) No condition on coeff. matrix

22   The local discretization error in Euler method is of the order of
                (a) h (b) h^2 (c) h^3 (d) h^4

23   In Simpson $3/8^{th}$ rule, the number of subintervals should be taken as the

        multiple of

        (a) 2   (b) 4   (c) 3   (d) none of these

24   The function whose first difference is e^x is

        (a) $e^x / e^x - 1$ (b) e^x (c) e^{-x} (d) none of these

25   Given a real valued function $g(x) = -\sqrt{x}$. Can the fixed point iteration

        algorithm be applied to find the root?

        (YES   /   NO)

26   If ω is first infinite ordinal, then $(\omega + 1)Z$ is
            (a) $\omega Z + Z$ (b) $(\omega + \omega) + 1$ (c) ωZ (d) $2^Z \omega$

27   In the ordinal space $[0, \Omega]$ every countable subset of $[0, \Omega)$ has an upper
        bound in
            (a) $[0, \Omega)$ (b) $\{\Omega\}$ (c) $[0, \omega)$ (d) $[\Omega, \omega)$

28   Let N_x be the collection of all neighborhoods of x in a space Y. Let
        $N_x \subset F$ where F is a filter. Then
            (a) $F \succ x$ (b) $F \to x$ (c) $N_x = F$ (d) None of these

29   A filter F in a space X clusters at x iff
            (a) $F \to x$ (b) $x \in \bigcap_{A \in F} A$ (c) $x \in \bigcap_{A \in F} \overline{A}$ (d) None of these

30   Let X be $T_{3\frac{1}{2}}$ and $\hat{X}$ any compactification of X. If $\beta(X)$ is stone-cech

        compactification of X. Then
            (a) $\hat{X}$ is homeomorphic to $\beta(X)$.
            (b) $\hat{X}$ is homeomorphic to a subspace of $\beta(X)$
            (c) $\hat{X}$ is homeomorphic to a quotient space of $\beta(X)$
            (d) (a), (b) are correct and (c) is false.

31   Let $Y \subset X$ and $i : Y \to X$ be the inclusion map such that I is continuous. Then the topology of Y

    a.   is relative topology with respect to X.

    b.   contains the relative topology of Y.

    c.   Is contained in the relative topology of Y.

    d.   None of these.

32   How many conjugacy classes are there in S_5

    (a) 5   (B) 6   (c) 7   (d) 8

33   The number of ring homomorphisms from $Z \to Q$ is

    (a) 1   (b) 2   (c) 4   (d) Infinite

34   Which of the following statements is correct

    (a) Every normal series is a composition series.

    (b) Every group with a composition series is a solvable group.

    (c) Every finite group has a composition series.

    (d) Every finite group is solvable.

35   The number of ideals in $Z_8 \times Z_9$ is

    (a) 10   (b) 11   (c) 12   (d) 13

36   Let F be a field of order 3 and V be a 2-dimentional vector space over F. If v is a nonzero element in V, then the number of ways to extend v into a bases of V is

    (a) 6   (b) 7   (c) 8   (d) 9

37   Groups of the which of the orders are solvable

    (a) 6   (b) 8   (c) 30   (d) All of these

38   How many subspaces of dimension 1 are there in a two dimensional vector space over R

    (a) 2   (b) 3   (c) 4   (d) None of these

39   If E is measurable and $m^{*}(E) = \infty$, then there exists a sequence $\{E_n\}$ of disjoint measurable sets such that $m^{*}(E_n) = \infty \, \forall n$ and

    (a) $E \subset \bigcup_{n=1}^{\infty} E_n$ (b) $E = \bigcup_{n=1}^{\infty} E_n$ (c) $E \supset \bigcup_{n=1}^{\infty} E_n$ (d) All of these

40   Which of these is measurable set?

    (a) Open interval in R (b) Closed interval in R

    (c) Union of two intervals   (d) All of these

41   Monotone convergence theorem holds only for

    (a) increasing sequence   (b) decreasing sequence

    (c) all sequences   (d) convergent sequences

42   Holder's inequality is valid for

    (a) $p > 1, q > 1$ (b) $p = 1 = q$ (c) $p < 1, q < 1$ (d) none of these

43 Let g be integrable on $[0, 1]$ and M is a constant such that $\left|\int fg\right| \leq M\|f\|_p$ for every bounded measurable function f, then

    (a) $g \in L^q$ (b) $\|g\|_q \leq M$ (c) both of these (d) none of these

44 N is non zero Banach space then $\{x \in N / \|x\| = 1\}$ is

    (a) Complete   (b) Open   (c) Closed   (d) All of these

45 M is a subspace of normed linear space N and f is a functional on M. Then f can be extended to a functional f_0 on N such that

    (a) $f_0(x) = f(x) \ \forall \ x \in M$ (b) $\|f_0\| = \|f\|$

    (c) Both (a) and (b) are true   (d) none of these

46 N and N' are normed linear spaces. T is continuous linear transformation of N into N' then norm of T is defined as

    (a) $\|T\| = \sup\{\|T(x)\| / \|x\| \leq 1\}\}$ (b) $\|T\| = \sup\{\|T(x)\| / \|x\| = 1\}$

    (c) $\|T\| = \sup\left\{\dfrac{\|Tx\|}{\|x\|} / x \in N, x \neq 0\right\}$ (d) all of these

47 Which of the following is parallelogram law

    (a) $\|x+y\|^2 - \|x-y\|^2 + i\|x+iy\|^2 - i\|x-iy\|^2 = 4(x,y)$

    (b) $\|x+y\|^2 + \|x-y\|^2 = 2\|x\|^2 + 2\|y\|^2$

    (c) $\|x+y\|^2 = (\|x\| + \|y\|)^2$

    (d) none of these

48 The set of all unitary operators on Hilbert space H form a

    (a) Group   (b) Field   (c) Ring   (d) Semi-group

49 If curvature and torsion of a curve are constant, then the curve is
(a) Parabolic cylinder (b) Cylindrical helix (c) Circular helix (d) space curve

50 If C represents a space curve and C_1, the locus of the centre of the osculating sphere then the product of torsions of C and C_1 at corresponding points is

    (a) zero   (b) constant (c) product of curvatures (d) 5

51 The meridians and parallels of a surface of revolutions are
(a) parallel (b) its line of curvature (c) perpendicular (d) parametric curves

52 The edge of regression of the polar developable is the locus of the centre of

    (a) osculating spheres   (b) circular helix

    (c) circle of curvature   (d) paraboloid

53 Two asymptotic lines through any point on a surface have

    (a) same curvature   (b) opposite curvature

    (c) same torsion   (d) opposite torsion

54   The set of all complex numbers is not a
        (a) open set   (b) closed set
        (c) perfect set   (d) bounded set

55   Suppose $0 < \delta < \pi$, $f(x) = 1$ if $|x| \le \delta$, $f(x) = 0$ if $\delta < |x| \le \pi$, and

        $f(x + 2\pi) = f(x)$ for all x. Then $\sum\limits_{n=1}^{\infty} \dfrac{\sin(n\delta)}{n}$ is

        (a) $\dfrac{\pi}{2}$ (b) $-\dfrac{\pi}{2}$ (c) 0   (d) none of these

56      $\int\limits_{\pi}^{2\pi} \sin x \, d(\cos x)$ is

        (a) $-\dfrac{\pi}{2}$ (b) $\dfrac{\pi}{2}$ (c) 0   (d) 1

57   Let a_{ij} be the number in ith row and jth column of the array

$$
\begin{array}{cccc}
-1 & 0 & 0 & 0 \\
\frac{1}{2} & -1 & 0 & 0 \\
\frac{1}{4} & \frac{1}{2} & -1 & 0 \\
\frac{1}{8} & \frac{1}{4} & \frac{1}{2} & -1
\end{array}
$$

....

        Then $\sum\limits_{i}\sum\limits_{j} a_{ij}$ equals to

        (a) 0   (b) 1   (c) -1   (d) none of these

58   Suppose f is real, continuously differentiable function on $[a,b]$,

        $f(a) = f(b) = 0$, then $\int\limits_{a}^{b} xf(x)f'(x)dx$ equals to

        (a) 1/2   (b) 1   (c) 0   (d) -1/2

59   In Ascoli's lemma, the family F consists of uniformly bounded and
        .... Functions
        (a) Continuous   (b) Equicontinuous
        (c) Differentiable   (d) None of these

60   In Picard's Lindelof theorem, the function $f(t,x)$ of $x' = f(t,x)$ should be
        .... in the domain D of (t,x) plane
        (a) Continuous   (b) Differentiable
        (c) Bounded and continuous   (d) None of these

61   Non trivial solution of $\dfrac{d^2 y}{dx^2} + \lambda y = 0, y(0) = y(\pi) = 0$ exists for

        (a) $\lambda > 0$ (b) $\lambda < 0$ (c) $\lambda = 0$ (d) only for complex λ

62   A homogeneous equation $L_n x = 0$ is self adjoint if the adjoint operator $\overline{L_n}$
        is equal to
        (a) L_n (b) $-L_n$ (c) L_n^{2} (d) none of these

63 $\dfrac{d^2y}{dx^2} - \dfrac{dy}{dx} + f(y) = 0$ is .... equation

(a) parabolic   (b) elliptic   (c) hyperbolic   (d) none of these

64 In any undirected graph, the sum of degrees of all the vertices
(a) must be even   (b) is twice the number of edges
(c) must be odd   (d) both (a) and (b)

65 Preorder traversal is nothing but ....
(a) depth of first order   (b) breadth of first order
(c) topological order   (d) liner order

66 The principle of duality is defined as
(a) $\leq$ is replaced by $\geq$
(b) lub is replaced by glb
(c) All properties not altered when $\leq$ is replaced by $\geq$
(d) All properties not altered when $\leq$ is replaced by $\geq$ other then 0 and 1 element.

67 Which of the following set is connected?
(a) $\{z : -1 < |z| < 1\} \cup \{2\}$ (b) $\{z : -\pi/4 < \arg z \leq \pi/4\}$
(c) $\{z : |z| < 1\} \cup \{z : |z - 1| < 1\}$ (d) none of these

68 The function $f(z) = z \operatorname{Re} z$ is differentiable at
(a) origin   (b) $z = 1$ (c) $z = -1$ (d) none of these

69 $Log(z - 1)$ is analytic except on the
(a) Half plane $x \leq 0$ (b) Half line $x \leq 0 (y = 1)$
(c) Half line $x \leq 0 (y > 1)$ (d) none of these

70 The value of $\displaystyle\int_{|z|=1} \dfrac{\sinh z}{z^2(z-2)} dz$ is

(a) π (b) πi (c) $\pi^2 i$ (d) none of these

71 The transformation which transforms the upper half plane into the lower half plane is

(a) $w = \bar{z}$ (b) $w = \dfrac{z-i}{z+i}$ (c) $w = 1/z$ (d) none of these

72 The number of zeros counting multiplicities of the polynomial
$z^5 + 3z^3 + z^2 + 1 = 0$ inside the circle $|z| = 2$ is

(a) 0   (b) 2   (c) 3   (d) 5

73 The value of $\displaystyle\int_\gamma \cot z \, dz, z = 4e^{4i\theta}, -\pi \leq \theta \leq \pi$ is

(a) $6\pi i$ (b) $-6\pi i$ (c) $24\pi i$ (d) $16\pi i$

74 The domain of convergence of the series $\displaystyle\sum n^2 \left(\dfrac{z^2+1}{1+i}\right)^n$ is

(a) $|z+1| < 2$ (b) $|z^2+1| < \sqrt{2}$
(c) $|z^2+1| < 2$ (d) $|z+1| < \sqrt{2}$

75   The efficiency index of an iterative method is defined by
        (a) $p^{1/n}$ (b) p^n (c) p^{n^2} (d) none of these

76   An analytic function whose only singularities in the finite complex plane
        are .... is called a meromorphic function.

        (a) essential (b) isolated   (c) non isolated   (d) none of these

77   If the height of a tree is 10, the highest level of the tree is
        (a) 10   (b) 9   (c) 5   (d) 1

78   In Fibonacci search technique with n=5 and $L_0 = 1$, the measure of
        effectiveness is
            (a) 0.001   (b) 0.01   (c) 0.1   (d) 0.125.

79   The function $F(x) = 3x_1^2 - 2x_2^2 + x_3^2$ is
        (a) positive definite   (b) positive semi-definite
        (c) negative definite   (d) indefinite.

80   Which one of the following is not a deterministic model
            (a) Linear programming problem   (b) Transportation problem
            (c) CPM   (d) PERT.

81   In a balanced transportation problem with m sources and n destinations,
        the number of dual constraints will be
            (a) m+n   (b) m+n+1
            (c) m+n-1   (d) mn

82   Addition of variable and deletion of a constraint simultaneously to a LPP
            (a) disturbs feasibility   (b) disturbs optimality
            (c) may disturb both feasibility and optimality (d) none of these.

83   If the dual LPP has an unbounded solution, then the primal problem
        has
            (a) Optimal solution   (b) infeasible solution
            (c) unbounded solution   (d) none of these.

84   Let min $f(X) = C^T X$, $AX \geq b$, $X \geq 0$ be a primal LPP. Suppose X_0 and
        Y_0 are the primal and dual feasible. Then
            (a) $C^T X_0 \leq b^T Y_0$ (b) $C^T X_0 \geq b^T Y_0$
            (c) $C^T X_0 = b^T Y_0$ (d) None of these.

85   The set $P_F \setminus A$, where A is the set of all vertices of P_F is
            (a) convex set   (b) not a convex set
            (c) may or may not be convex   (d) none of these

86   Consider the set $S = \{(x_1, x_2); x_2^2 \leq x_1\}$. Then S has

            (a) no vertex   (b) finite number of vertices
            (c) infinite number of vertices. (d) None of these

87   Application of dual simplex method requires that availability vector b must satisfy

           (a) $b \geq 0$ (b) $b \leq 0$

           (c) no restriction of (a) and (b) type   (d) $b = 0$

88   The Boolean function $\overline{xy} + xy + \overline{x}y$ is equivalent to

           (a) $\overline{x} + \overline{y}$ (b) $x + y$ (c) $x + \overline{y}$ (d) $\overline{x} + y$

89

     The solution of $\dfrac{dx}{dt} = x^2, x(1) = -1 \ldots \ldots on[0,1]$

           (a) can be continued   (b) cannot be continued

           (c) exists   (d) none of these

90   Let $A = N \times N$, and the set $F_{\{m,n\}} = \{(x,y) : x, y \in R \text{ and } |x| > m, |y| > n\}$ then $\bigcap F_{\{m,n\}}$ equals to

           (a) 0   (b) 1   (c) $\bigcap F_{\{m,n\}}$ does not have finite intersection property

           (d) none of these