S-08

Entrance Examination, 2010
M.Sc. (Statistics-OR)

Hall Ticket No.	

Time : 2 hours
Max. Marks. 75

Part A : 25 marks
Part B : 50 marks

Instructions

1. **Write your Hall Ticket Number in the OMR answer sheet given to you. Also write the Hall Ticket number in the space provided above.**
2. There is negative marking.
3. Answers are to be marked on the OMR answer sheet .
4. Please read the instructions carefully before marking your answers on the OMR answer sheet.
5. Hand over the question paper booklet and the OMR answer sheet at the end of the examination.
6. No additional sheets will be provided. Rough work can be done in the question paper itself/space provided at the end of the booklet.
7. Calculators are not allowed.
8. There are a total of 50 questions in Part A and Part B together.
9. The appropriate answer should be coloured in either a blue or black ball point or sketch pen. DO NOT USE A PENCIL.

PART A

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- Find the correct answer and mark it on the OMR answer sheet.
- A right answer gets **1 mark** and a wrong answer gets **−0.33 mark**.

1. 5 red balls(all alike) and 4 blue balls(all alike) are to be placed in two bags numbered 1 and 2. The number of ways in which this can be done is

   [A] 2048.

   [B] 512.

   [C] 24.

   [D] 30.

2. A company wants to reduce its sales force by dismissing 4 of its 10 sales people. In how many different ways can this be done?

   [A] 40.

   [B] 210.

   [C] 5040.

   [D] 151,200.

3. One page of a 500 page book is defective. It is decided to find the page, the first 250 pages are non defective, the probability that the defective page is one of the pages numbered $251, 252, \ldots, 300$

   [A] is $\frac{1}{10}$.

   [B] is $\frac{1}{2}$.

   [C] is $\frac{1}{5}$.

   [D] cannot be determined from the data given.

4. Two boxes contain 100 balls which are marked $1, 2, \ldots, 100$. One ball is picked up from each box, what is the probability that the two balls bear the same number?

   [A] $\frac{1}{100}$

   [B] $\frac{2}{100}$

   [C] $\frac{1}{100^2}$

   [D] $\frac{2}{100^2}$

5. 4 girls G_1, G_2, G_3, G_4 and 4 boys B_1, B_2, B_3, B_4 are randomly arranged in a row. The probabilities that all the girls are seated in such a way that there is no boy between any two girls is

[A] $\frac{5!}{8!}$.
[B] $\frac{4!5!}{8!}$.
[C] $\frac{4!4!}{8!}$.
[D] $\frac{2.4!.4!}{8!}$.

6. Which measure of central tendency among the measures given below is influenced by all the values in the data set?

[A] Mean.
[B] Median.
[C] Mode.
[D] Midrange.

7. If there is data on shoe sizes for adult males in India which are given as 5,6,7,8 or 9, which of the following is the most suitable measure of central tendency?

[A] Arithmetic mean.
[B] Geometric mean.
[C] Median.
[D] Mode.

8. The distribution of marks of 100 students in a class is as follows

Marks	0-30	30-45	45-55	55-80	80-95	95-100
Frequency	0	25	55	12	6	2

The distribution of marks is

[A] uniform.
[B] symmetric but not uniform.
[C] positively skewed.
[D] negatively skewed.

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9. Marks of 6 students in two courses C_1 and C_2 are given below.

			Students			
	1	2	3	4	5	6
Course C_1	40	41	42	43	44	45
Course C_2	40	41.5	41.5	43.5	43.5	45

Let (M_i, s_i) denote the mean and the standard deviation of the marks in $C_i, i = 1, 2$ respectively. It can be seen that

[A] $M_1 = M_2$ and $s_1 > s_2$.

[B] $M_1 > M_2$ and $s_1 > s_2$.

[C] $M_1 < M_2$ and $s_1 < s_2$.

[D] $M_1 = M_2$ and $s_1 < s_2$.

10. The random variable X follows a Binomial distribution with parameters $n = 10$ and $p = 0.5$, i.e $X \sim B(10, 0.5)$. Define another random variable Y as $Y = 10 - X$, then $E(X - Y)$ is equal to

[A] 5.

[B] 10.

[C] -5.

[D] 0.

11. A coin for which probability of heads showing up upon tossing is $\frac{1}{3}$ is tossed 30 times. Which of the following statements is correct?

[A] 5 heads are more likely to show up than 8 heads.

[B] 15 heads are more likely to show up than any other number of heads.

[C] 10 heads are more likely to show up than any other number of heads.

[D] 20 heads are more likely to show up than 15 heads.

12. X is a discrete random variable for which $E(X) = V(X)$. Then X is distributed as

[A] a Poisson random variable.

[B] a Binomial random variable.

[C] a Negative Binomial random variable.

[D] None of the above.

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13. The random variable X has exponential distribution with mean 5. $P(X > 5)$ is

[A] e^{-5}.

[B] $e^{\frac{-1}{5}}$.

[C] $\frac{1}{5}e^{-1}$.

[D]. e^{-1}.

14. The correlation coefficient between two random variables X and Y is $\frac{1}{2}$. Define two new random variables U and V as $U = 2X + 7$ and $V = 9Y + 12$. The correlation coefficient between U and V is

[A] 0.

[B] $\frac{1}{4}$.

[C] $\frac{1}{2}$.

[D] 1.

15. Given below are measurements for six individuals on two random variables X and Y

X	2	5	6	8	9	12
Y	4.1	10.1	12.1	16.1	18.1	24

We can say that the correlation coefficient between X and Y is

[A] 0.

[B] almost -1.

[C] almost $+1$.

[D] between $\frac{1}{3}$ and $\frac{1}{2}$.

16. Non constant random variables X and Y are distributed in such a way that $V(X + Y) = V(X - Y)$, then

[A] X and Y are necessarily independently distributed.

[B] $cov(X, Y) = 0$.

[C] $V(X)$ must be equal to $V(Y)$.

[D] X and Y are necessarily identically distributed.

17. In order to draw a representative sample from a population, it is divided into at least two subgroups and then a random sample is drawn from each of the subgroups. What is this type of sampling scheme called?

[A] Aggregate sampling.

[B] Cluster sampling.

[C] Cross-sectional sampling.

[D] Stratified sampling.

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18. Based on a random sample of size n from the $N(\mu, 1)$ population to test $H_0 : \mu = \mu_o$ against $H_1 : \mu > \mu_o$, the most powerful level α test is $\bar{X}_n > a_n$ where $\bar{X}_n$ is the sample mean. Which is the following statements is correct about the behavior of a_n as n changes for the same level of significance?

[A] a_n remains the same no matter what n is.

[B] a_n decreases as n increases.

[C] a_n increases as n increases.

[D] a_n increases up to some n, say n_0 and then decreases.

19. Let X be a random variable taking values $0, 1, 2, 3, a$ with probabilities $\frac{1}{6}, \frac{1}{3}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}$ respectively. If $E(X)$ is $\frac{11}{6}$, then a is

[A] 6.

[B] 7.

[C] 4.

[D] 5.

20. T is an unbiased estimator for a parameter θ and $V(T) = \theta^2 + \theta$. An unbiased estimator for θ^2 is

[A] $\frac{T(T-1)}{2}$.

[B] $\frac{T^2+T}{2}$.

[C] $\frac{T^2}{2}$.

[D] none of the above.

21. Let $\mathbf{x}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\mathbf{x}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ and $\mathbf{x}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ be vectors in $\mathbb{R}^3$. Let $\mathbf{x}_4 = \alpha_1\mathbf{x}_1 + \alpha_2\mathbf{x}_2 + \alpha_3\mathbf{x}_3$ where $\alpha_i \in \mathbb{R}$ for $i = 1, 2, 3$. The choice of α_1, α_2 and α_3 for which the set of vectors $\mathbf{x}_1, \mathbf{x}_2$ and $\mathbf{x}_4$ do not form a basis is

[A] $\alpha_1 = \frac{1}{2}, \alpha_2 = \frac{1}{2}, \alpha_3 = \frac{1}{2}$.

[B] $\alpha_1 = \frac{1}{2}, \alpha_2 = \frac{1}{2}, \alpha_3 = 0$.

[C] $\alpha_1 = \frac{-1}{2}, \alpha_2 = \frac{1}{2}, \alpha_3 = \frac{1}{2}$.

[D] $\alpha_1 = \frac{1}{2}, \alpha_2 = \frac{-1}{2}, \alpha_3 = \frac{1}{2}$.

22. Consider the following linear programming problem

$$\begin{array}{lrcl} \textit{Maximize} & 5x_1 + 7x_2 & & \\ \textit{Subject to} & 2x_1 + 3x_2 & \leq & 33 \\ & 3x_1 + x_2 & \leq & 25 \\ & 4x_1 + x_2 & \leq & 32 \\ & x_1, x_2 & \geq & 0 \end{array}$$

Which of the following is the optimal solution to the problem ?

[A] $x_1 = 6, x_2 = 7$.

[B] $x_1 = 10, x_2 = 9$.

[C] $x_1 = 7, x_2 = 4$.

[D] $x_1 = 8, x_2 = 0$.

23. A is a 4×4 real non singular matrix, a matrix B is obtained in the following way.
The first row of B is the sum of the first and second rows of A,
The second row of B is the sum of the second and third rows of A,
The third and fourth rows of B are the third and fourth rows of A respectively.
The rank of B is

[A] 1.

[B] 2.

[C] 3.

[D] 4.

24. The series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n!}$

[A] diverges.

[B] does not exist.

[C] converges to a positive number.

[D] converges to 0.

25. The negation of the statement '$a_j \geq a_{j+1}$ for all $j = 1, 2, \ldots$' is

[A] $a_j < a_{j+1}$ for all $j = 1, 2, \ldots$.

[B] $a_j < a_{j+1}$ for some $j = 1, 2, \ldots$.

[C] $a_j < a_{j+1}$ for at most one $j = 1, 2, \ldots$.

[D] $a_j = a_{j+1}$ for all $j = 1, 2, \ldots$.

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Part B

- Find the correct answer and mark it on the OMR answer sheet.
- A right answer gets **2 marks** and a wrong answer gets **−0.66 mark**.

26. At least one of the events A or B will certainly occur, the probability of both occurring is $\frac{1}{4}$. The probability that at most one of them will occur

    [A] cannot be determined from the given information.

    [B] is 0.

    [C] is $\frac{1}{4}$.

    [D] is $\frac{1}{2}$.

27. A fair coin is thrown 5 times , let E_i denote the event that the i^{th} and the $(i+2)^{th}$ throw show heads for $i = 1, 2, 3$ respectively. Which of the following statements is correct?

    [A] E_1, E_3 are not independent.

    [B] E_1, E_2 are not independent.

    [C] E_2, E_3 are not independent.

    [D] E_1, E_2, E_3 are jointly independent.

28. B is an event for which $0 < P(B) < 1$. Two events A_1 and A_2 are equiprobable when B occurs as well as when B does not occur. Then

    [A] A_1 and A_2 are necessarily independent.

    [B] A_1 and A_2 are equiprobable.

    [C] A_1 and A_2 are certainly mutually exclusive.

    [D] A_1 always occurs when A_2 occurs.

29. The probability that the difference between two numbers drawn without replacement from $1, 2\ldots, 20$ is at least 10 is in the interval

    [A] $(0, \frac{1}{4}]$.

    [B] $(\frac{1}{4}, \frac{1}{2}]$.

    [C] $(\frac{1}{2}, \frac{3}{4}]$.

    [D] $(\frac{1}{4}, 1)$.

30. 10 red balls (all alike) and 10 blue balls(all alike) are randomly placed in a row. The probability that there are more red balls among the first 10 is

[A] $\dfrac{\binom{10}{5}^2}{\binom{20}{10}}$

[B] $\displaystyle\sum_{x=5}^{10} \dfrac{\binom{10}{x}^2}{\binom{20}{10}}$

[C] $\displaystyle\sum_{x=6}^{10} \dfrac{\binom{10}{x}}{\binom{20}{10}}$

[D] $\displaystyle\sum_{x=6}^{10} \dfrac{\binom{10}{x}^2}{\binom{20}{10}}$

31. There are as many males as females in a population. $\frac{3}{4}^{th}$ of the females believe in a rumour while $\frac{7}{8}^{th}$ of the males believe it. A person is chosen randomly from this population and is found to believe the rumour. The probability that the person chosen was female is

[A] $\frac{3}{13}$.
[B] $\frac{4}{13}$.
[C] $\frac{5}{13}$.
[D] $\frac{6}{13}$.

32. The range of three positive integers is 3 and their variance is 2, then

[A] exactly two of them are equal.
[B] the three integers are distinct.
[C] all are equal.
[D] this can never happen.

33. The median marks of all the twenty girls of a class is 12 and that of all the eighty boys of the same class is 6. Which of the following statements on the median of all the 100 students of the class is correct?

[A] It is certainly more than 6.
[B] It can be less than 6.
[C] It is certainly less than 12.
[D] It is certainly more than 12.

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34. Anand tosses a fair coin(say $C1$) and stops when a head appears. Bharath does the same experiment with a fair coin (say $C2$). The probability that both of them tossed their respective coins the same number of times is

[A] $\frac{1}{2}$.

[B] $\frac{1}{3}$.

[C] $\frac{1}{4}$.

[D] $\frac{2}{3}$.

35. The expected value of a random variable whose probability density function is

$$f_X(x) = \begin{cases} cx^4 & -3 < x < 3 \\ 0 & otherwise \end{cases}$$

[A] is 0.

[B] does not exist.

[C] is 1.

[D] is 2.

36. The expected value of a uniformly distributed random variable X is 5 and $P(X \leq 6) = \frac{2}{3}$, the variance of X

[A] is $\frac{25}{3}$.

[B] is $\frac{25}{12}$.

[C] is 3.

[D] cannot be determined from the given information.

37. Consider a distribution of data that is not necessarily bell-shaped. According to Chebyshev's theorem, the minimum percentage of data values lying within ±1.6 standard deviations from the mean would be

[A] 37.5%

[B] 39.0625%

[C] 60.9375%

[D] 62.5%

38. $X_1 \sim B(n, p_1), X_2 \sim B(n, p_2); E(X_1) > E(X_2)$ and $V(X_1) < V(X_2)$, then it is true that

[A] $p_1 > p_2 > \frac{1}{2}$

[B] $p_2 > p_1 > \frac{1}{2}$

[C] $\frac{1}{2} > p_2 > p_1$

[D] $p_1 > \frac{1}{2} > p_2$

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39. A fair die is rolled till 6 shows up. the first 16 throws were unsuccessful(i.e. 6 did not show up in any of the first 16 throws), the probability that the first 6 will show up in the 19th throw is

[A] $(\frac{5}{6})^{19}$

[B] $(\frac{5}{6})^{18}\frac{1}{6}$

[C] $(\frac{5}{6})^{3}$

[D] $(\frac{5}{6})^{2}\frac{1}{6}$

40. Ashok and Ravi play a game in which a toss a fair coin alternately and who ever gets the first heads is the winner and the game stops. If Ashok takes the first toss, the expected number of tosses that Ashok took till the game stopped is

[A] $\frac{4}{3}$.

[B] $\frac{4}{2}$.

[C] 1.

[D] 4.

41. The number of scratches in a compact disc is a Poisson random variable with mean 1. The probability that there are two scratches in two randomly selected compact discs is

[A] $2e^{-1}$.

[B] e^{-2}.

[C] $2e^{-2}$.

[D] e^{-1}.

42. $X_1, \ldots, X_n$ is a random sample from the $U(0, \theta), \theta > 0$ population. Let $X_{(n)} = max\,(X_1, \ldots, X_n)$.

[A] $X_{(n)}$ is an unbiased as well as a sufficient estimator of θ.

[B] $X_{(n)}$ is neither an unbiased nor a sufficient estimator of θ.

[C] $X_{(n)}$ is an unbiased but not a sufficient estimator of θ.

[D] $X_{(n)}$ is not unbiased but is a sufficient estimator of θ.

43. The sum of 10 independent observations of a Bernoulli random variable X for which $P(X = 1) = p$ and $P(X = 0) = 1 - p$; $0 < p < 1$ was 7. The maximum likelihood estimate of p if $p \in \left\{\frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}\right\}$ is

[A] $\frac{1}{4}$.

[B] $\frac{1}{2}$.

[C] $\frac{1}{3}$.

[D] $\frac{2}{3}$.

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44. The value of $\sum_{m=0}^{n} \binom{n}{m} \left(\sum_{i=0}^{m} (-1)^i \binom{m}{i} \right)$ is

[A] 0.

[B] 1.

[C] 2.

[D] 3.

45. Consider the function $f(x) = e^{5x-x^2}, -\infty < x < \infty$, which of the following statements is correct?

[A] Its maximum value is 1 which is attained at $x = 0$.

[B] Its maximum value is greater than 1 and is attained at x which is positive.

[C] It has more than one maxima.

[D] It is an increasing function.

46. Define the function f as follows

$$f(x) = \begin{cases} \frac{x^2-x}{|x|} & x \neq 0 \\ 1 & x = 0 \end{cases}$$

then

[A] f is continuous everywhere.

[B] f is differentiable at $x = 0$.

[C] f is continuous everywhere except at $x = 0$.

[D] f attains a finite maximum.

47. T_1 and T_2 are two unbiased estimators based on the same sample for a parameter θ but $V(T_1) < V(T_2)$. If the observed values of T_1 and T_2 are a_1 and a_2 respectively, we should

[A] choose a_1 to estimate θ because it is very close to θ.

[B] choose a_2 to estimate θ because it is closer to θ than a_1 is.

[C] choose a_1 to estimate θ because it is more likely to be close to θ than a_2.

[D] Toss a fair coin, choose a_1 if it shows heads otherwise choose a_2 to estimate θ.

48. Suppose the hypothesis $H_o : \mu = \mu_0$ is accepted against $H_1 : \mu > \mu_0$ at α level of significance, (where μ is the mean of a Normal random variable with variance 2). This means that

[A] the data shows strong evidence in favour of H_0.

[B] H_0 can be accepted even if the data shows strong evidence in favour of $H_2 : \mu < \mu_0$.

[C] the data shows strong evidence in favour of H_1.

[D] the sample mean is equal to μ_0.

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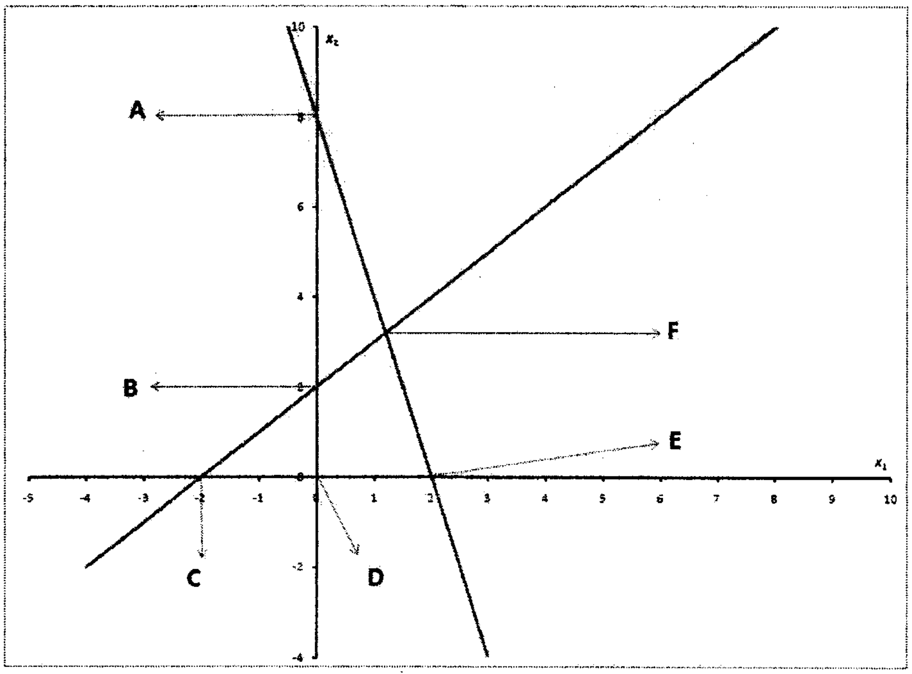

Figure 1: Figure for Problems 49 and 50

The next two questions are based on the following linear programming problem and the figure given in Figure (1).

$$\begin{array}{lrcl} Min & x_1 + x_2 & & \\ s.t. & 4x_1 + x_2 & \leq & 8 \\ & -x_1 + x_2 & \leq & 2 \\ & x_1, x_2 & \geq & 0 \end{array}$$

49. The feasible region of the linear programming problem is given by

[A] the triangle ABF.

[B] the triangle ADE.

[C] the triangle FCE.

[D] the quadilateral $FBDE$.

50. If constraint $4x_1 + x_2 \leq 8$ is dropped , which of the following is true?

[A] The feasible region is unbounded and the optimal solution occurs at point D.

[B] The feasible region does not change.

[C] The feasible region is unbounded and the optimal value is $-\infty$.

[D] The feasible region does not change and the optimal solution is -2.