Graduate Aptitude Test in Engineering

Notations :

1.Options shown in green color and with ✔ icon are correct.

2.Options shown in red color and with ✖ icon are incorrect.

Question Paper Name: MA: MATHEMATICS 1st Feb shift2
Number of Questions: 65
Total Marks: 100.0

Wrong answer for MCQ will result in negative marks, (-1/3) for 1 mark Questions and (-2/3) for 2 marks Questions.

General Aptitude

Number of Questions: 10
Section Marks: 15.0

Q.1 to Q.5 carry 1 mark each & Q.6 to Q.10 carry 2 marks each.

Question Number : 1 Question Type : MCQ

Choose the appropriate word/phrase, out of the four options given below, to complete the following sentence:

Apparent lifelessness ____________________ dormant life.

(A) harbours (B) leads to (C) supports (D) affects

Options :

1. ✔ A
2. ✖ B
3. ✖ C
4. ✖ D

Question Number : 2 Question Type : MCQ

Fill in the blank with the correct idiom/phrase.

That boy from the town was a ____________ in the sleepy village.

(A) dog out of herd (B) sheep from the heap
(C) fish out of water (D) bird from the flock

Options :

1. ✖ A
2. ✖ B
3. ✔ C
4. ✖ D

Question Number : 3 Question Type : MCQ

Choose the statement where underlined word is used correctly.

(A) When the teacher eludes to different authors, he is being <u>elusive</u>.
(B) When the thief keeps eluding the police, he is being <u>elusive</u>.
(C) Matters that are difficult to understand, identify or remember are <u>allusive</u>.
(D) Mirages can be <u>allusive</u>, but a better way to express them is illusory.

Options :

1. ✖ A
2. ✔ B
3. ✖ C
4. ✖ D

Question Number : 4 Question Type : MCQ

Tanya is older than Eric.
Cliff is older than Tanya.
Eric is older than Cliff.

If the first two statements are true, then the third statement is:

(A) True
(B) False
(C) Uncertain
(D) Data insufficient

Options :

1. ✖ A
2. ✔ B
3. ✖ C
4. ✖ D

Question Number : 5 Question Type : MCQ

Five teams have to compete in a league, with every team playing every other team exactly once, before going to the next round. How many matches will have to be held to complete the league round of matches?

(A) 20 (B) 10 (C) 8 (D) 5

Options :

1. ✖ A
2. ✔ B
3. ✖ C
4. ✖ D

Question Number : 6 Question Type : MCQ

Select the appropriate option in place of underlined part of the sentence.

<u>Increased productivity necessary</u> reflects greater efforts made by the employees.

(A) Increase in productivity necessary
(B) Increase productivity is necessary
(C) Increase in productivity necessarily
(D) No improvement required

Options :

1. ✖ A
2. ✖ B
3. ✔ C
4. ✖ D

Question Number : 7 Question Type : MCQ

Given below are two statements followed by two conclusions. Assuming these statements to be true, decide which one logically follows.

Statements:

I. No manager is a leader.
II. All leaders are executives.

Conclusions:

I. No manager is an executive.
II. No executive is a manager.

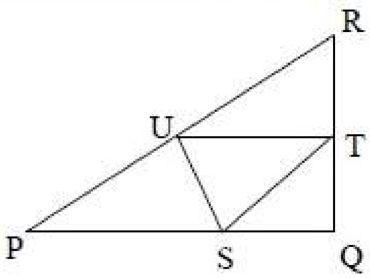
(A) Only conclusion I follows.
(B) Only conclusion II follows.
(C) Neither conclusion I nor II follows.
(D) Both conclusions I and II follow.

Options :

1. ✖ A
2. ✖ B
3. ✔ C
4. ✖ D

Question Number : 8 Question Type : NAT

In the given figure angle Q is a right angle, PS:QS = 3:1, RT:QT = 5:2 and PU:UR = 1:1. If area of triangle QTS is 20 cm^2, then the area of triangle PQR in cm^2 is _______.



Correct Answer :

280

Question Number : 9 Question Type : MCQ

Right triangle PQR is to be constructed in the xy - plane so that the right angle is at P and line PR is parallel to the x-axis. The x and y coordinates of P, Q, and R are to be integers that satisfy the inequalities: $-4 \leq x \leq 5$ and $6 \leq y \leq 16$. How many different triangles could be constructed with these properties?

(A) 110 (B) 1,100 (C) 9,900 (D) 10,000

Options :

1. ✖ A
2. ✖ B
3. ✔ C
4. ✖ D

Question Number : 10 Question Type : MCQ

A coin is tossed thrice. Let X be the event that head occurs in each of the first two tosses. Let Y be the event that a tail occurs on the third toss. Let Z be the event that two tails occur in three tosses. Based on the above information, which one of the following statements is TRUE?

(A) X and Y are not independent (B) Y and Z are dependent
(C) Y and Z are independent (D) X and Z are independent

Options :

1. ✖ A
2. ✔ B
3. ✖ C
4. ✖ D

Mathematics

Number of Questions:	55
Section Marks:	85.0

Q.11 to Q.35 carry 1 mark each & Q.36 to Q.65 carry 2 marks each.

Question Number : 11 Question Type : NAT

Let $T : \mathbb{R}^4 \rightarrow \mathbb{R}^4$ be a linear map defined by

$$T(x, y, z, w) = (x + z, 2x + y + 3z, 2y + 2z, w).$$

Then the rank of T is equal to _________

Correct Answer :

3

Question Number : 12 Question Type : NAT

Let M be a 3×3 matrix and suppose that 1, 2 and 3 are the eigenvalues of M. If

$$M^{-1} = \frac{M^2}{\alpha} - M + \frac{11}{\alpha} I_3$$

for some scalar $\alpha \neq 0$, then α is equal to ___________

Correct Answer :

6

Question Number : 13 Question Type : NAT

Let M be a 3×3 singular matrix and suppose that 2 and 3 are eigenvalues of M. Then the number of linearly independent eigenvectors of $M^3 + 2M + I_3$ is equal to __________

Correct Answer :

3

Question Number : 14 Question Type : NAT

Let M be a 3×3 matrix such that $M \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 6 \\ -3 \\ 0 \end{pmatrix}$ and suppose that $M^3 \begin{pmatrix} 1 \\ -1/2 \\ 0 \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}$ for some $\alpha, \beta, \gamma \in \mathbb{R}$. Then $|\alpha|$ is equal to _______

Correct Answer:

27

Question Number : 15 Question Type : MCQ

Let $f: [0, \infty) \to \mathbb{R}$ be defined by

$$f(x) = \int_0^x \sin^2(t^2)\, dt.$$

Then the function f is

(A) uniformly continuous on $[0, 1)$ but NOT on $(0, \infty)$
(B) uniformly continuous on $(0, \infty)$ but NOT on $[0, 1)$
(C) uniformly continuous on both $[0, 1)$ and $(0, \infty)$
(D) neither uniformly continuous on $[0, 1)$ nor uniformly continuous on $(0, \infty)$

Options :

1. ✖ A
2. ✖ B
3. ✔ C
4. ✖ D

Question Number : 16 Question Type : NAT

Consider the power series $\sum_{n=0}^{\infty} a_n z^n$, where $a_n = \begin{cases} \frac{1}{3^n} & \text{if } n \text{ is even} \\ \frac{1}{5^n} & \text{if } n \text{ is odd.} \end{cases}$

The radius of convergence of the series is equal to ___________

Correct Answer :

3

Question Number : 17 Question Type : NAT

Let $C = \{ z \in \mathbb{C} : |z - i| = 2\}$. Then $\frac{1}{2\pi} \oint_C \frac{z^2 - 4}{z^2 + 4} dz$ is equal to _____________

Correct Answer :

-2

Question Number : 18 Question Type : NAT

Let $X \sim B(5, \frac{1}{2})$ and $Y \sim U(0,1)$. Then $\frac{P(X+Y \leq 2)}{P(X+Y \geq 5)}$ is equal to ____________

Correct Answer:

6

Question Number : 19 Question Type : NAT

Let the random variable X have the distribution function

$$F(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{x}{2} & \text{if } 0 \leq x < 1 \\ \frac{3}{5} & \text{if } 1 \leq x < 2 \\ \frac{1}{2} + \frac{x}{8} & \text{if } 2 \leq x < 3 \\ 1 & \text{if } x \geq 3. \end{cases}$$

Then $P(2 \leq X < 4)$ is equal to ____________

Correct Answer:

0.4

Question Number : 20 Question Type : NAT

Let X be a random variable having the distribution function

$$F(x) = \begin{cases} 0 & \text{if} \quad x < 0 \\ \frac{1}{4} & \text{if } 0 \le x < 1 \\ \frac{1}{3} & \text{if } 1 \le x < 2 \\ \frac{1}{2} & \text{if } 2 \le x < \frac{11}{3} \\ 1 & \text{if} \quad x \ge \frac{11}{3}. \end{cases}$$

Then $E(X)$ is equal to __________

Correct Answer :

2.25

Question Number : 21 Question Type : MCQ

In an experiment, a fair die is rolled until two sixes are obtained in succession. The probability that the experiment will end in the fifth trial is equal to

(A) $\frac{125}{6^5}$ (B) $\frac{150}{6^5}$ (C) $\frac{175}{6^5}$ (D) $\frac{200}{6^5}$

Options :

1. ✖ A
2. ✖ B
3. ✔ C
4. ✖ D

Question Number : 22 Question Type : MCQ

Let $x_1 = 2.2,\ x_2 = 4.3,\ x_3 = 3.1,\ x_4 = 4.5,\ x_5 = 1.1$ and $x_6 = 5.7$ be the observed values of a random sample of size 6 from a $U(\theta - 1,\ \theta + 4)$ distribution, where $\theta \in (0, \infty)$ is unknown. Then a maximum likelihood estimate of θ is equal to

(A) 1.8 (B) 2.3 (C) 3.1 (D) 3.6

Options :

1. ✔ A
2. ✖ B
3. ✖ C
4. ✖ D

Question Number : 23 Question Type : MCQ

Let $\Omega = \{(x,y) \in \mathbb{R}^2 | \, x^2 + y^2 < 1\}$ be the open unit disc in $\mathbb{R}^2$ with boundary $\partial\Omega$. If $u(x,y)$ is the solution of the Dirichlet problem

$$u_{xx} + u_{yy} = 0 \qquad \text{in } \Omega$$
$$u(x,y) = 1 - 2\,y^2 \qquad \text{on } \partial\Omega,$$

then $u\left(\frac{1}{2}, 0\right)$ is equal to

(A) -1 (B) $\frac{-1}{4}$ (C) $\frac{1}{4}$ (D) 1

Options :

1. ✖ A
2. ✖ B
3. ✔ C
4. ✖ D

Question Number : 24 Question Type : NAT

Let $c \in \mathbb{Z}_3$ be such that $\frac{\mathbb{Z}_3[X]}{\langle X^3 + c\,X + 1\rangle}$ is a field. Then c is equal to __________

Correct Answer :

2

Question Number : 25 Question Type : MCQ

Let $V = C^1[0,1]$, $X = (\,C[0,1], \|\quad\|_\infty)$ and $Y = (\,C[0,1], \|\quad\|_2)$. Then V is

(A) dense in X but NOT in Y
(B) dense in Y but NOT in X
(C) dense in both X and Y
(D) neither dense in X nor dense in Y

Options :

1. ✖ A
2. ✖ B
3. ✔ C
4. ✖ D

Question Number : 26 Question Type : NAT

Let $T : \; (\,C[0,1], \|\quad\|_\infty) \to \mathbb{R}$ be defined by $T(f) = \int_0^1 2xf(x)\,dx$ for all $f \in C[0,1]$. Then $\|T\|$ is equal to __________

Correct Answer :

1

Question Number : 27 Question Type : MCQ

Let τ_1 be the usual topology on $\mathbb{R}$. Let τ_2 be the topology on $\mathbb{R}$ generated by $\mathcal{B} = \{[a,b) \subset \mathbb{R} : -\infty < a < b < \infty\}$. Then the set $\{x \in \mathbb{R} : 4\sin^2 x \leq 1\} \cup \left\{\frac{\pi}{2}\right\}$ is

(A) closed in $(\mathbb{R}, \tau_1)$ but NOT in $(\mathbb{R}, \tau_2)$
(B) closed in $(\mathbb{R}, \tau_2)$ but NOT in $(\mathbb{R}, \tau_1)$
(C) closed in both $(\mathbb{R}, \tau_1)$ and $(\mathbb{R}, \tau_2)$
(D) neither closed in $(\mathbb{R}, \tau_1)$ nor closed in $(\mathbb{R}, \tau_2)$

Options :

1. ✖ A
2. ✖ B
3. ✔ C
4. ✖ D

Question Number : 28 Question Type : MCQ

Let X be a connected topological space such that there exists a non-constant continuous function $f : X \to \mathbb{R}$, where $\mathbb{R}$ is equipped with the usual topology. Let $f(X) = \{f(x) : x \in X\}$. Then

(A) X is countable but $f(X)$ is uncountable
(B) $f(X)$ is countable but X is uncountable
(C) both $f(X)$ and X are countable
(D) both $f(X)$ and X are uncountable

Options :

1. ✖ A
2. ✖ B
3. ✖ C
4. ✔ D

Question Number : 29 Question Type : MCQ

Let d_1 and d_2 denote the usual metric and the discrete metric on $\mathbb{R}$, respectively. Let $f : (\mathbb{R}, d_1) \to (\mathbb{R}, d_2)$ be defined by $f(x) = x,\ x \in \mathbb{R}$. Then

(A) f is continuous but f^{-1} is NOT continuous
(B) f^{-1} is continuous but f is NOT continuous
(C) both f and f^{-1} are continuous
(D) neither f nor f^{-1} is continuous

Options :

1. ✖ A
2. ✔ B
3. ✖ C
4. ✖ D

Question Number : 30 Question Type : NAT

If the trapezoidal rule with single interval $[0, 1]$ is exact for approximating the integral $\int_0^1 (x^3 - c\,x^2)dx$, then the value of c is equal to ________

Correct Answer :

1.5

Question Number : 31 Question Type : NAT

Suppose that the Newton-Raphson method is applied to the equation $2x^2 + 1 - e^{x^2} = 0$ with an initial approximation x_0 sufficiently close to zero. Then, for the root $x = 0$, the order of convergence of the method is equal to ________

Correct Answer :

1

Question Number : 32 Question Type : NAT

The minimum possible order of a homogeneous linear ordinary differential equation with real constant coefficients having $x^2 \sin(x)$ as a solution is equal to __________

Correct Answer:

6

Question Number : 33 Question Type : MCQ

The Lagrangian of a system in terms of polar coordinates (r, θ) is given by

$$L = \frac{1}{2}\, m\, \dot{r}^2 + \frac{1}{2}\, m\left(\dot{r}^2 + r^2\dot{\theta}^2\right) - m\, g\, r\,(1 - \cos(\theta)),$$

where m is the mass, g is the acceleration due to gravity and $\dot{s}$ denotes the derivative of s with respect to time. Then the equations of motion are

(A) $2\,\ddot{r} = r\,\dot{\theta}^2 - g\,(1 - \cos(\theta)),\ \frac{d}{dt}(r^2\dot{\theta}) = -g\,r\,\sin(\theta)$

(B) $2\,\ddot{r} = r\,\dot{\theta}^2 + g\,(1 - \cos(\theta)),\ \frac{d}{dt}(r^2\dot{\theta}) = -g\,r\,\sin(\theta)$

(C) $2\,\ddot{r} = r\,\dot{\theta}^2 - g\,(1 - \cos(\theta)),\ \frac{d}{dt}(r^2\dot{\theta}) = g\,r\,\sin(\theta)$

(D) $2\,\ddot{r} = r\,\dot{\theta}^2 + g\,(1 - \cos(\theta)),\ \frac{d}{dt}(r^2\dot{\theta}) = g\,r\,\sin(\theta)$

Options :

1. ✔ A
2. ✖ B
3. ✖ C
4. ✖ D

Question Number : 34 Question Type : NAT

If $y(x)$ satisfies the initial value problem

$$(x^2 + y)dx = x\,dy, \qquad y(1) = 2,$$

then $y(2)$ is equal to __________

.
Correct Answer:
6

Question Number : 35 Question Type : NAT

It is known that Bessel functions $J_n(x)$, for $n \geq 0$, satisfy the identity

$$e^{\frac{x}{2}\left(t-\frac{1}{t}\right)} = J_0(x) + \sum_{n=1}^{\infty} J_n(x)\left(t^n + \frac{(-1)^n}{t^n}\right)$$

for all $t > 0$ and $x \in \mathbb{R}$. The value of $J_0\left(\frac{\pi}{3}\right) + 2\sum_{n=1}^{\infty} J_{2n}\left(\frac{\pi}{3}\right)$ is equal to ________

Correct Answer :
1

Question Number : 36 Question Type : MCQ

Let X and Y be two random variables having the joint probability density function

$$f(x,y) = \begin{cases} 2 & \text{if } 0 < x < y < 1 \\ 0 & \text{otherwise.} \end{cases}$$

Then the conditional probability $P\left(X \leq \frac{2}{3} \mid Y = \frac{3}{4}\right)$ is equal to

(A) $\frac{5}{9}$ (B) $\frac{2}{3}$ (C) $\frac{7}{9}$ (D) $\frac{8}{9}$

Options :

1. ✖ A
2. ✖ B
3. ✖ C
4. ✔ D

Question Number : 37 Question Type : NAT

Let $\Omega = (0,1]$ be the sample space and let $P(\cdot)$ be a probability function defined by

$$P((0,x]) = \begin{cases} \frac{x}{2} & \text{if } 0 \leq x < \frac{1}{2} \\ x & \text{if } \frac{1}{2} \leq x \leq 1. \end{cases}$$

Then $P\left(\left\{\frac{1}{2}\right\}\right)$ is equal to ________

Correct Answer :
0.25

Question Number : 38 Question Type : NAT

Let X_1, X_2 and X_3 be independent and identically distributed random variables with $E(X_1) = 0$ and $E(X_1^2) = \frac{15}{4}$. If $\psi : (0, \infty) \to (0, \infty)$ is defined through the conditional expectation

$$\psi(t) = E(X_1^2 \mid X_1^2 + X_2^2 + X_3^2 = t), \ \ t > 0,$$

then $E(\psi((X_1 + X_2)^2))$ is equal to ___________

Correct Answer :

2.5

Question Number : 39 Question Type : NAT

Let $X \sim \text{Poisson}(\lambda)$, where $\lambda > 0$ is unknown. If $\delta(X)$ is the unbiased estimator of $g(\lambda) = e^{-\lambda}(3\lambda^2 + 2\lambda + 1)$, then $\sum_{k=0}^{\infty} \delta(k)$ is equal to ___________

Correct Answer :

9

Question Number : 40 Question Type : NAT

Let $X_1, \ldots, X_n$ be a random sample from $N(\mu, 1)$ distribution, where $\mu \in \{0, \frac{1}{2}\}$. For testing the null hypothesis $H_0 : \mu = 0$ against the alternative hypothesis $H_1 : \mu = \frac{1}{2}$, consider the critical region

$$R = \left\{(x_1, x_2, \ldots, x_n) : \sum_{i=1}^{n} x_i > c\right\},$$

where c is some real constant. If the critical region R has size 0.025 and power 0.7054, then the value of the sample size n is equal to ___________

Correct Answer :

25

Question Number : 41 Question Type : MCQ

Let X and Y be independently distributed central chi-squared random variables with degrees of freedom $m \ (\geq 3)$ and $n \ (\geq 3)$, respectively. If $E\left(\frac{X}{Y}\right) = 3$ and $m + n = 14$, then $E\left(\frac{Y}{X}\right)$ is equal to

(A) $\frac{2}{7}$ (B) $\frac{3}{7}$ (C) $\frac{4}{7}$ (D) $\frac{5}{7}$

Options :

1. ✖ A
2. ✖ B
3. ✖ C
4. ✔ D

Question Number : 42 Question Type : NAT

Let $X_1, X_2, \dots$ be a sequence of independent and identically distributed random variables with $P(X_1 = 1) = \frac{1}{4}$ and $P(X_1 = 2) = \frac{3}{4}$. If $\overline{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$, for $n = 1, 2, \dots$, then $\lim_{n\to\infty} P(\overline{X}_n \le 1.8)$ is equal to ___________

Correct Answer:

1

Question Number : 43 Question Type : MCQ

Let $u(x,y) = 2f(y)\cos(x - 2y)$, $(x,y) \in \mathbb{R}^2$, be a solution of the initial value problem

$$2u_x + u_y = u$$
$$u(x,0) = \cos(x).$$

Then $f(1)$ is equal to

(A) $\frac{1}{2}$ (B) $\frac{e}{2}$ (C) e (D) $\frac{3e}{2}$

Options :

1. ✖ A
2. ✔ B
3. ✖ C
4. ✖ D

Question Number : 44 Question Type : NAT

Let $u(x,t)$, $x \in \mathbb{R}$, $t \ge 0$, be the solution of the initial value problem

$$u_{tt} = u_{xx}$$
$$u(x,0) = x$$
$$u_t(x,0) = 1.$$

Then $u(2,2)$ is equal to _________

Correct Answer:

4

Question Number : 45 Question Type : NAT

Let $W = \text{Span}\left\{\frac{1}{\sqrt{2}}(0,0,1,1), \frac{1}{\sqrt{2}}(1,-1,0,0)\right\}$ be a subspace of the Euclidean space $\mathbb{R}^4$. Then the square of the distance from the point $(1,1,1,1)$ to the subspace W is equal to _________

Correct Answer :

2

Question Number : 46 Question Type : NAT

Let $T : \mathbb{R}^4 \to \mathbb{R}^4$ be a linear map such that the null space of T is

$$\{(x, y, z, w) \in \mathbb{R}^4 : x + y + z + w = 0\}$$

and the rank of $(T - 4\, I_4)$ is 3. If the minimal polynomial of T is $x(x-4)^{\alpha}$, then α is equal to ________

Correct Answer :

1

Question Number : 47 Question Type : MCQ

Let M be an invertible Hermitian matrix and let $x, y \in \mathbb{R}$ be such that $x^2 < 4y$. Then

(A) both $M^2 + x\,M + y\,I$ and $M^2 - x\,M + y\,I$ are singular
(B) $M^2 + x\,M + y\,I$ is singular but $M^2 - x\,M + y\,I$ is non-singular
(C) $M^2 + x\,M + y\,I$ is non-singular but $M^2 - x\,M + y\,I$ is singular
(D) both $M^2 + x\,M + y\,I$ and $M^2 - x\,M + y\,I$ are non-singular

Options :

1. ✖ A
2. ✖ B
3. ✖ C
4. ✔ D

Question Number : 48 Question Type : MCQ

Let $G = \{e, x, x^2, x^3, y, xy, x^2y, x^3y\}$ with $o(x) = 4, o(y) = 2$ and $xy = yx^3$. Then the number of elements in the center of the group G is equal to

(A) 1 (B) 2 (C) 4 (D) 8

Options :

1. ✖ A
2. ✔ B
3. ✖ C
4. ✖ D

Question Number : 49 Question Type : NAT

The number of ring homomorphisms from $\mathbb{Z}_2 \times \mathbb{Z}_2$ to $\mathbb{Z}_4$ is equal to __________

Correct Answer:

1

Question Number : 50 Question Type : MCQ

Let $p(x) = 9x^5 + 10x^3 + 5x + 15$ and $q(x) = x^3 - x^2 - x - 2$ be two polynomials in $\mathbb{Q}[x]$. Then, over $\mathbb{Q}$,

(A) $p(x)$ and $q(x)$ are both irreducible
(B) $p(x)$ is reducible but $q(x)$ is irreducible
(C) $p(x)$ is irreducible but $q(x)$ is reducible
(D) $p(x)$ and $q(x)$ are both reducible

Options :

1. ✖ A
2. ✖ B
3. ✔ C
4. ✖ D

Question Number : 51 Question Type : NAT

Consider the linear programming problem

$$\begin{aligned} &\text{Maximize } 3x + 9y, \\ &\text{subject to} \quad 2y - x \le 2 \\ &\qquad\qquad\quad 3y - x \ge 0 \\ &\qquad\qquad\quad 2x + 3y \le 10 \\ &\qquad\qquad\quad x, y \ge 0. \end{aligned}$$

Then the maximum value of the objective function is equal to ______

Correct Answer:

24

Question Number : 52 Question Type : MCQ

Let $S = \{ (x, \sin\frac{1}{x}) : 0 < x \le 1 \}$ and $T = S \cup \{(0,0)\}$. Under the usual metric on $\mathbb{R}^2$,

(A) S is closed but T is NOT closed
(B) T is closed but S is NOT closed
(C) both S and T are closed
(D) neither S nor T is closed

Options :

1. ✖ A
2. ✖ B
3. ✖ C
4. ✔ D

Question Number : 53 Question Type : MCQ

Let $H = \left\{ (x_n) \in \ell_2 : \sum_{n=1}^{\infty} \frac{x_n}{n} = 1 \right\}$. Then H

(A) is bounded
(B) is closed
(C) is a subspace
(D) has an interior point

Options :

1. ✖ A
2. ✔ B

3. ✖ C

4. ✖ D

Question Number : 54 Question Type : MCQ

Let V be a closed subspace of $L^2[0,1]$ and let $f, g \in L^2[0,1]$ be given by $f(x) = x$ and $g(x) = x^2$. If $V^{\perp} = \text{Span}\{f\}$ and Pg is the orthogonal projection of g on V, then $(g - Pg)(x)$, $x \in [0,1]$, is

(A) $\frac{3}{4}x$ (B) $\frac{1}{4}x$ (C) $\frac{3}{4}x^2$ (D) $\frac{1}{4}x^2$

Options :

1. ✔ A

2. ✖ B

3. ✖ C

4. ✖ D

Question Number : 55 Question Type : NAT

Let $p(x)$ be the polynomial of degree at most 3 that passes through the points $(-2,12), (-1,1)$, $(0,2)$ and $(2,-8)$. Then the coefficient of x^3 in $p(x)$ is equal to __________

Correct Answer :

-2

Question Number : 56 Question Type : NAT

If, for some $\alpha, \beta \in \mathbb{R}$, the integration formula

$$\int_0^2 p(x)dx = p(\alpha) + p(\beta)$$

holds for all polynomials $p(x)$ of degree at most 3, then the value of $3(\alpha - \beta)^2$ is equal to ______

Correct Answer :

4

Question Number : 57 Question Type : NAT

Let $y(t)$ be a continuous function on $[0, \infty)$ whose Laplace transform exists. If $y(t)$ satisfies

$$\int_0^t (1 - \cos(t - \tau))\, y(\tau)\, d\tau = t^4,$$

then $y(1)$ is equal to ________

Correct Answer :

28

Question Number : 58 Question Type : NAT

Consider the initial value problem

$$x^2y'' - 6\,y = 0, \qquad y(1) = \alpha,\ y'(1) = 6.$$

If $y(x) \to 0$ as $x \to 0^+$, then α is equal to ___________

Correct Answer :

2

Question Number : 59 Question Type : MCQ

Define $f_1, f_2 \colon [0,1] \to \mathbb{R}$ by

$$f_1(x) = \sum_{n=1}^{\infty} \frac{x\ \sin(n^2x)}{n^2} \quad \text{and} \quad f_2(x) = \sum_{n=1}^{\infty} x^2(1-x^2)^{n-1}.$$

Then

(A) f_1 is continuous but f_2 is NOT continuous
(B) f_2 is continuous but f_1 is NOT continuous
(C) both f_1 and f_2 are continuous
(D) neither f_1 nor f_2 is continuous

Options :

1. ✔ A
2. ✖ B
3. ✖ C
4. ✖ D

Question Number : 60 Question Type : NAT

Consider the unit sphere $S = \{(x,y,z) \in \mathbb{R}^3 \colon x^2 + y^2 + z^2 = 1\}$ and the unit normal vector $\hat{n} = (x,y,z)$ at each point (x,y,z) on S. The value of the surface integral

$$\iint_S \{ (\frac{2\,x}{\pi} + \sin(y^2))\, x + (e^z - \frac{y}{\pi})\, y + (\frac{2\,z}{\pi} + \sin^2 y)\, z\}\ d\sigma$$

is equal to ________

Correct Answer :

4

Question Number : 61 Question Type : NAT

Let $D = \{(x,y) \in \mathbb{R}^2 \colon 1 \leq x \leq 1000,\ 1 \leq y \leq 1000\}$. Define

$$f(x,y) = \frac{x\,y}{2} + \frac{500}{x} + \frac{500}{y}.$$

Then the minimum value of f on D is equal to _________

Correct Answer:
150

Question Number : 62 Question Type : MCQ

Let $\mathbb{D} = \{ z \in \mathbb{C} : |z| < 1 \}$. Then there exists a non-constant analytic function f on $\mathbb{D}$ such that for all $n = 2, 3, 4, \ldots$

(A) $f\left(\frac{\sqrt{-1}}{n}\right) = 0$

(B) $f\left(\frac{1}{n}\right) = 0$

(C) $f\left(1 - \frac{1}{n}\right) = 0$

(D) $f\left(\frac{1}{2} - \frac{1}{n}\right) = 0$

Options :

1. ✖ A
2. ✖ B
3. ✔ C
4. ✖ D

Question Number : 63 Question Type : NAT

Let $\sum_{n=-\infty}^{\infty} a_n z^n$ be the Laurent series expansion of $f(z) = \frac{1}{2z^2 - 13z + 15}$ in the annulus $\frac{3}{2} < |z| < 5$. Then $\frac{a_1}{a_2}$ is equal to __________

Correct Answer:
5

Question Number : 64 Question Type : NAT

The value of $\frac{i}{4-\pi} \int_{|z|=4} \frac{dz}{z \cos(z)}$ is equal to ___________

Correct Answer :
2

Question Number : 65 Question Type : MCQ

Suppose that among all continuously differentiable functions $y(x)$, $x \in \mathbb{R}$, with $y(0) = 0$ and $y(1) = \frac{1}{2}$, the function $y_0(x)$ minimizes the functional

$$\int_0^1 \left(e^{-(y'-x)} + (1+y)y'\right)dx.$$

Then $y_0\left(\frac{1}{2}\right)$ is equal to

(A) 0

(B) $\frac{1}{8}$

(C) $\frac{1}{4}$

(D) $\frac{1}{2}$

Options :

1. ✖ A

2. ✔ B
3. ✖ C
4. ✖ D