

ENTRANCE TEST FOR ADMISSION 2000

Integrated Ph.D

Mathematical Sciences

Day & Date : Sunday 14th May 2000

Time : 1.30 p.m. to 4.30 p.m.



**INDIAN INSTITUTE OF SCIENCE
BANGALORE**

INSTRUCTIONS

- The question paper is in two parts: Part A and Part B. Part A carries 30 marks and Part B carries 70 marks.
- Part A comprises 30 multiple choice questions each carrying 1 mark. Four possible answers are provided for each question. Select the correct answer by marking (✓) against (a), (b), (c) or (d) on the answer script exactly as given below.
For example, Question: $2 + 2 =$ Answer: (a) 0 (b) 2 (✓) 4 (d) 8.
Answer all questions from Part A.
- Part B comprises 8 questions. Answer any 5 questions. Each question carries 14 marks.
- All answers must be written in the answer book and *not on the question paper*.

MATHEMATICAL SCIENCES

Part A

1. If $x + 1 :: 1 + x$ then x is

- (a) a rational number
- (b) an imaginary number
- (c) an irrational number
- (d) none of the above

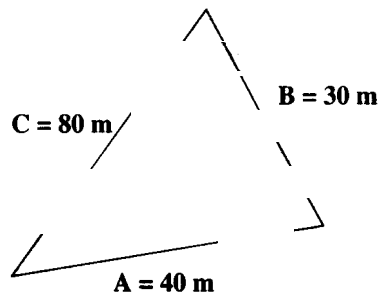


FIG. 1.

2. A surveyor measures the dimensions of a triangular piece of land (assumed to be flat) and claims that they are as shown in Figure 1. This means that the surveyor has most likely

- (a) underestimated side C
- (b) overestimated sides A and B
- (c) underestimated sides A and B, or overestimated C, or both
- (d) measured the sides correctly.

3. If x, y are nonzero real numbers, then $x^2 + xy + y^2$ is
- (a) always positive
 - (b) always negative
 - (c) zero
 - (d) sometimes positive, sometimes negative.
4. If $\tan \theta + \cos \theta = m$ and $\tan \theta - \cos \theta = n$, then the value of $\sin 2\theta$ is
- (a) $\frac{mn}{4}$
 - (b) $\frac{m^2n^2}{4}$
 - (c) $\frac{(m+n)^2(m-n)}{4}$
 - (d) $\frac{(m-n)^2(m+n)}{4}$
5. The area of the smallest region bounded by the curves $y = |x|$ and $x^2 + y^2 = 4$ is
- (a) π
 - (b) 2π
 - (c) $\frac{3\pi}{4}$
 - (d) $\frac{3\pi}{2}$
6. Let \hat{i} , \hat{j} , and \hat{k} be the unit vectors along the usual x , y , and z axes. A unit vector perpendicular to $\hat{i} + \hat{j} + \hat{k}$ and lying in the xy - plane is
- (a) $\hat{j} - \hat{k}$
 - (b) $\frac{1}{\sqrt{2}}(\hat{j} - \hat{k})$
 - (c) $\hat{i} - \hat{j}$
 - (d) $\frac{1}{\sqrt{2}}(\hat{i} - \hat{j})$

7. Two spherical planets A and B have the same density, but the acceleration due to gravity at the surface of A is $1/6$ of that at the surface of B. This means that the ratio of the radius of A to that of B is

- (a) 1
- (b) $1/6$
- (c) $1/36$
- (d) $1/216$

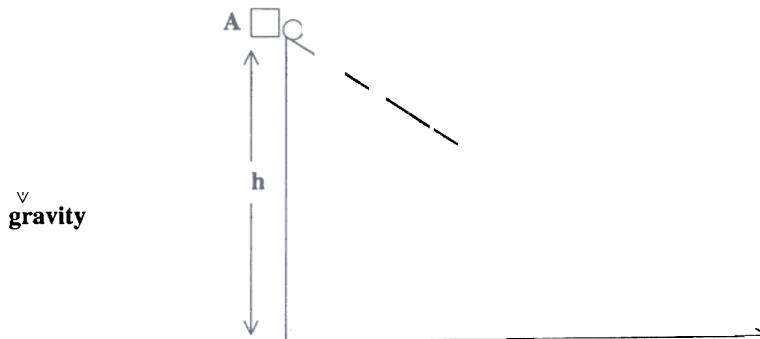


FIG. 2.

8. In Figure 2, object A is dropped vertically downwards with initial velocity 0, while object B rolls without slipping down the inclined plane. The masses of A and B are equal, and both start from the top of the inclined plane with initial velocity zero. Ignore rolling friction and air resistance. Which of the following is correct?

- (a) B has higher total kinetic energy than A when they reach the bottom, because B rotates *and* translates;
- (b) A will have higher total kinetic energy than B when they reach the bottom
- (c) The final kinetic energy of each of the objects depends on its shape;
- (d) none of the above.

9. A large plastic balloon has a volume of 300 m^3 when completely filled. Approximately how many cubic metres of helium gas, at temperature 27° C and standard atmospheric pressure, should it be filled with if it is to be completely full when it reaches its designed altitude where the pressure is $1/3$ of an atmosphere, and the temperature is -53° C ?
- (a) 140 m^3
 - (b) 19600 m^3
 - (c) 14 m^3
 - (d) 1.4 m^3

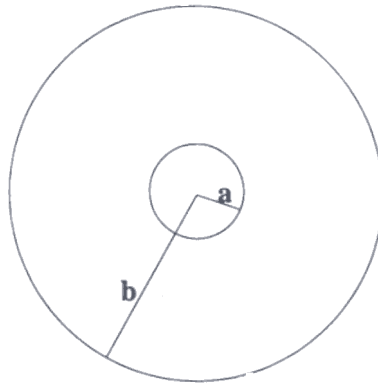


FIG. 3.

10. An iron washer (Figure 3) has an outer radius b and an inner radius a . If heated,
- (a) a increases and b decreases
 - (b) b increases and a decreases
 - (c) Both a and b increase
 - (d) None of the above.

- 11 1 cc of oil is spread on a surface to form a circular film of uniform thickness with no holes. Based on what you know about the sizes of molecules, which of the following is a reasonable estimate for the maximum possible radius of the film?
- (a) ∞
 - (b) 1 cm to 10 cm
 - (c) 1000 m to 3000 m
 - (d) 10 m to 60 m

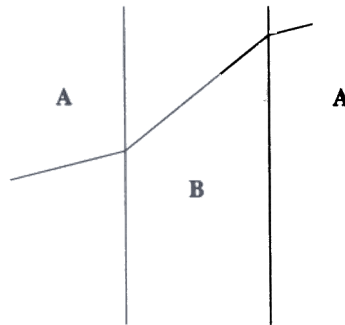


FIG. 4.

12. Figure 4 shows a ray of light passing from a medium A through a medium B and back into A. Which of the following is consistent with the figure?
- (a) A is air, B is glass;
 - (b) A is vacuum, B is diamond;
 - (c) A is air, B is water;
 - (d) A is glass, B is air.

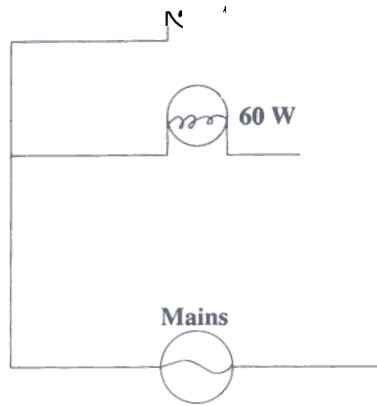


FIG. 5.

13. Figure 5 shows a 40 W and a 60 W light bulb connected to the mains (voltage fixed

- (a) glows more brightly
- (b) glows less brightly
- (c) does not glow
- (d) glows with unchanged brightness.

14. Two metal spheres of the same radius, with initial charges Q_1 and Q_2 attract each other. After they come into contact, it is observed that they repel each other. This means

- (a) $Q_1 \times Q_2 < 0$, $Q_1 + Q_2 \neq 0$
- (b) $Q_1 \times Q_2 > 0$, $Q_1 + Q_2 \neq 0$
- (c) $Q_1 \times Q_2 > 0$, $Q_1 + Q_2 = 0$
- (d) $Q_1 \times Q_2 < 0$, $Q_1 + Q_2 = 0$

15. How many unique types of hydrogen atoms are present in hexa-2,4-diene?
- (a) 2;
 - (b) 3;
 - (c) 4;
 - (d) 5
16. During the sodium fusion test, nitrogen in an organic compound gets converted to
- (a) cyanide;
 - (b) cyanate;
 - (c) nitrogen gas;
 - (d) ammonia.
17. The entropy change ΔS associated with a spontaneous endothermic process satisfies
- (a) $\Delta S = 0$;
 - (b) $\Delta S < 0$;
 - (c) $\Delta S > 0$;
 - (d) $\Delta S > \Delta H$
18. In a first order chemical reaction, the concentration of the reactant decreases from 1.0 mol lit⁻¹ to 0.25 mol lit⁻¹ in 100 hours. The half-life of the reaction is
- (a) 50 hours;
 - (b) 100 hours;
 - (c) 200 hours;
 - (d) 75 hours.
19. Although F is more electronegative than Cl, HF is a weaker acid than HCl because
- (a) HF dimerizes;
 - (b) F is larger than Cl;
 - (c) the hydrogen bonding is stronger in HCl;
 - (d) Cl has the higher electron affinity.

20. For a chemical reaction at equilibrium:
- (a) a catalyst would shift the position of the equilibrium;
 - (b) a catalyst would increase the rates of forward and backward reactions;
 - (c) a catalyst would affect only the forward reaction;
 - (d) a catalyst would slow down the backward reaction.
21. Which of the following complexes will show paramagnetic behaviour?
- (a) $\text{Ni}(\text{CO})_4$;
 - (b) $\text{K}_2\text{Cr}_2\text{O}_7$;
 - (c) KMnO_4 ;
 - (d) $\text{K}_3[\text{Fe}(\text{CN})_6]$.
22. The coordination numbers of calcium and fluorine in CaF_2 (Fluorite) structure are respectively
- (a) 8 and 8;
 - (b) 6 and 6;
 - (c) 8 and 4;
 - (d) 4 and 8.
23. If there are 5 different bases in DNA and the genetic code consists of 4 bases per codon, the number of codons possible will be
- (a) 125
 - (b) 256
 - (c) 625
 - (d) 1024

24. The concentration of carbon dioxide has been increasing steadily in recent times due to human activities. How will this affect plant productivity?
- (a) Productivity will decrease because of CO₂ pollution.
 - (b) Productivity will increase because of higher CO₂ levels.
 - (c) CO₂ will not change productivity.
 - (d) Difficult to predict the outcome.
25. Humans and apes are similar at the DNA sequence level to the extent of
- (a) 50%
 - (b) 75%
 - (c) 90%
 - (d) > 90%
26. The major protein in hair is
- (a) keratin
 - (b) actin
 - (c) collagen
 - (d) fibrin
27. Plant cells can be distinguished from animal cells based on the fact that
- (a) Plant cells have a cell membrane which is absent in animal cells.
 - (b) Animal cells have mitochondria which are absent in plant cells.
 - (c) Plant cells have chloroplasts that animal cells do not have.
 - (d) Plant cells do not have a nucleus.
28. In a double stranded DNA molecule,
- (a) A+G = C+T
 - (b) A=T within each single strand
 - (c) G=C within each single strand
 - (d) All four bases are found in equal proportions

29. The strongest reason for believing that all life forms of today shared a common ancestor in the distant past is that
- (a) We all have ATP
 - (b) The genetic code is nearly universal
 - (c) Life can come only from life
 - (d) The alternative will be absurd.
30. Glycogen belongs to the category of compounds known as
- (a) Carbohydrate.
 - (b) Fat.
 - (c) Protein.
 - (d) nucleic acid.

MATHEMATICAL SCIENCES

PART B

1. Given a positive integer $m > 2$, show that there exist positive integers p and q such that $p < q$ and

$$\frac{1}{m} = \sum_{j=p+1}^q \frac{1}{j(j+1)}$$

2. Find the angles α, β, γ of a triangle if they satisfy the relation

$$\sin\left(\frac{\alpha-\beta}{2}\right) + \sin\left(\frac{\alpha-\gamma}{2}\right) + \sin\left(\frac{3\alpha}{2}\right) = \frac{3}{2}$$

3. Find all integers a for which the cubic equation

$$x^3 - x + a = 0$$

has three integer roots.

4. Prove that if $p > 1, x > 0$, $x^p - 1 \geq p(x - 1)$

5. Show that for any $x > 0$, $\int_0^x \frac{\sin t}{1+t} dt > 0$.

6. Find the radius of the circle which is obtained as a section of the sphere $x^2 + y^2 + z^2 = 9$ by the plane $x + y + z = 3$. Also find the equation of the cone with its vertex at $(0, 0, 0)$ and containing the above circle.

7. Find all the integers x in the set $\{1, 2, 3, \dots, 100\}$ such that $x^2 \equiv x \pmod{100}$

8. Solve the following equation for x

$$\begin{vmatrix} x & p & q & 1 \\ a & x & r & 1 \\ a & b & x & 1 \\ a & b & c & 1 \end{vmatrix} = 0.$$