Sardar Patel University
Model Question Paper
for Entrance Test for admission to Ph. D. (Mathematics)

Question	1	2	3	4	Total
Maximum Marks	10	40	25	25	100
Marks obtained					

Section – I [10]

Answer the following:

1) Who introduced the concept of a measurable set?
2) Show that $|re(z)| + |im(z)| \le \sqrt{2}|z|$.
3) Show that a measurable set is nearly an open set.
4) Show that the Lebesgue measure of Q is 0.
5) What is the dimension of R over Q? (Where R is the vector space of all real numbers and Q is the field of all rational numbers.)
6) Define Euclidean ring and give one example of it.
7) State the Division Algorithm for polynomials over a field.
8) Is it true that co-countable topology on the set of natural numbers is compact? Justify your answer
9) State Bessel's equation of order n.
10) What is degree of freedom of a simple harmonic oscillator?

Section – II [40]

Select the correct answer from the given options.

1) Suppose $z^2 + \alpha z + \beta = 0$ has a pair of complex conjugate roots. Then α and β are real with ______
   (a) $\alpha^2 < 4\beta$ (b) $4\alpha > \beta^2$ (c) $\alpha < 4\beta$ (d) $4\alpha > \beta$
2) The values of z for which the function w defined by w = u + iv; z = sinh u cos v + i cosh u sin v ceases to be analytic are _____
   (a) $\pm i$ (b) ± 1 (c) $1 \pm i$ (d) $-1 \pm i$
3) $w = e^{i\lambda}\frac{z-\alpha}{z\bar{\alpha}-1}$ maps $|z| \le 1$ onto _____.
   (a) $|w| \le 1|$ (b) $|w| \le |\lambda|$ (c) $|w| = 1$ (d) $|w| \ge |\lambda|$
4) Suppose z_1, z_2 and z_3 are vertices of an equilateral triangle. Then $\frac{1}{z_2-z_3}+\frac{1}{z_3-z_1}+\frac{1}{z_1-z_2} =$ ______.
   (a) 1 (b) 3 (c) 1/3 (d) none of these
5) The residue of $\frac{4z+3}{z(z-3)(z+2)}$ in $0 < |z| < 1$ at $z = 0$ is ________.
   (a) –1/2 (b) –1/4 (c) –1/3 (d) 0
6) z = 1 is a ________ of $\frac{z-2}{z+1} \sin\frac{1}{z-1}$.
   (a) isolated essential singularity
   (b) non-isolated essential singularity
   (c) removable singularity
   (d) pole

7) The number of zeros of $z^4 + 4(1+i)z + 1 = 0$ in the first quadrant is __________.

(a) 2 (b) 0 (c) 1 (d) none of these

8) The value of $\int_0^{\pi} \frac{1+2\cos\theta}{5+4\cos\theta}\, d\theta$ = ___________.

(a) 1 (b) π (c) $\pi/2$ (d) none of these

9) The value of $\int_{|z|=2} \frac{\cos z}{z^3}\, dz =$ _____________.

(a) $-i\pi$ (b) $i\pi$ (c) $2\pi i$ (d) $-2\pi i$

10) Let $f(x) = \begin{cases} 1 \; if \; x \; is \; rational \\ -1 \; if \; x \; is \; irrational \end{cases}$ $x \in [0,1]$. Then ___________.

(a) f is not Riemann Integrable
(b) f is continuous only at irrationals
(c) f is continuous everywhere
(d) f is Riemann Integrable

11) Let $f(x) = \frac{\tan(\pi[x-\pi])}{1+[x]^2}$. Then ________________________.

(a) f exists for all x
(b) f exists for all x but f' does not exist at some x
(c) f is discontinuous at some x
(d) f is continuous for all x but f'' does not exist at some x

12) Let $f(x) = \begin{cases} x \; if \; x \; is \; rational \\ -x \; if \; x \; is \; irrational \end{cases}$. Then ___________.

(a) f is discontinuous at all x except x = 0
(b) f is continuous
(c) f' is positive for all x
(d) none of these

13) The value of $\int_{-1}^{1} x - [x] dx$ is __________.

(a) 1 (b) 2 (c) 0 (d) 1/2

14) Let $f(x) = [x]^2 - [x^2]$. Then f is discontinuous at __________.

(a) all integers except 1 (b) all integers except 0
(c) all integers except 0 and 1 (d) all integers

15) Let F be a field of characteristic zero and A, B be n × n matrices over F (n > 1). Then AB – BA ____________.

(a) $\neq I$ (b) $= I$ (c) $= 0$ (d) $\neq 0$

16) Let X be an inner product space and x, y ∈ X be such that |<x, y>| = ||x|| ||y||. Then ______.

(a) x and y are linearly independent. (b) x and y are orthogonal.
(c) x and y are linearly dependent. (d) nothing can be said.

17) Let U and V be finite dimensional vector spaces of dimensions m and n respectively. Then dimension of the Hom(U, V) is _____.

(a) m + n (b) m n (c) m / n (d) m^n

18) Let V be a finite dimensional vector space over a field F and T ∈ A(V) be onto. Which of the following is not true?

(a) T is one to one (b) T is bijective (c) T is regular (d) T is singular

19) Which of the following is not complete in any norm?

(a) c_{00}, the space of all sequences of real numbers having finitely many nonzero terms.
(b) l^{∞}, the space of all bounded sequences of real numbers.
(c) C([0, 1]), the space of all real valued continuous functions defined on [0, 1].
(d) R^n, where R is the field of all real numbers.

20) Modified Euler's method is used to solve __________________

(a) Ordinary Differential equation (b) Partial Differential equation

(c) Quadratic Equation (d) Transcendental equation

21) ___________________ is used to solve f(x) = 0 numerically.

(a) Bisection method (b) Euler's method

(c) Gauss method (d) Simpson's method

22) Which of the following is not true?

(a) Every group of order 5 is abelian;

(b) Every cyclic group is abelian;

(c) If G is a finite group and p divides o(G), then G has a subgroup of order p;

(d) Every group has at least one subgroup.

23) Let f : G → H be a homomorphism with kernel K. Then

(a) K is a subgroup of H; (b) K is a normal subgroup of H;

(c) Both (i) and (ii) are true; (d) Neither (i) nor (ii) is true.

24) Which of the following is true?

(a) Caley's Theorem is only for abelian groups;

(b) Caley's Theorem is only for finite groups;

(c) Caley's Theorem is only for finite cyclic groups;

(d) Caley's Theorem is for all groups.

25) The polynomial $x^2 - 5$ is

(a) reducible over Q; (b) solvable by radicals over Q;

(c) not solvable by radicals over Q; (d) none.

26) Let I = {1, 2} and φ(x) = inf{|x-y| : y ∈ I} (x ∈ R). Then

(a) φ is not continuous at 1; (b) φ is not continuous at 2;

(c) φ is not continuous at 1 and 2; (d) None

27) Which of the following topological spaces is compact?

(a) an infinite set with co-finite topology;

(b) (0, 1] with usual topology;

(c) {1, 1/2 , 1/3, ….} with usual topology;

(d) An infinite set with co-countable topology.

28) The set of all limit points of Z in R with usual topology is

(a) Φ (b) Z (c) Q (d) R

29) Let R_l be the real line with lower limit topology. Then

(a) R_l is compact; (b) R_l is connected;

(c) R_l is separable; (d) R_l is metrizable.

30) Let Y be a subspace of a metric space X.

(a) If X is compact, then Y is compact;

(b) If X is complete, then Y is complete;

(c) If X is connected, then Y is connected;

(d) None.

31) If $I[y(x)] = \int_a^b f(x, y, y')dx$ has an extremum, then

(a) $\frac{\partial f}{\partial x} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0;$ (b) $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y}\right) = 0;$

(c) $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0;$ (d) $\frac{\partial f}{\partial y'} - \frac{d}{dx}\left(\frac{\partial f}{\partial y}\right) = 0;$

32) The extremum of $I[y(x)] = \int_1^2 (1 + x^2 y')y'dx$ with y(1) = 1 and y(2) = 0 is

(a) y(x) = 2/x – 1; (b) y(x) = 2/x – 2;

(c) y(x) = 2/x + x; (d) None.

33) The multi-integration $\int_a^x \int_a^u \int_a^t f(s)dsdtdu =$

(a) $\int_a^x f(z)dz$; (b) $\int_a^x (x-z)^2 f(z)dz$;

(c) $\frac{1}{2}\int_a^x (x-z)^2 f(z)dz$; (d) None.

34) Define f : $R^2 \to R$ as $f(x,y) = \sqrt{|xy|}$. Then

(a) f is differentiable at 0; (b) f is continuously differentiable at 0;

(c) Both (i) and (ii) are true; (d) Neither (i) nor (ii) is true.

35) If p is generalized momentum and q is generalized coordinate then $[p, q]$ = ________

(a) 1 (b) −2 (c) 2 (d) −1

36) Which of the following is 1-dimensional wave equation?

(a) $\frac{\partial u}{\partial t} = c^2 \frac{\partial u}{\partial x}$ (b) $\frac{\partial u}{\partial t} = 0$ (c) $\frac{\partial u}{\partial x} = c^2 \frac{\partial^2 u}{\partial x^2}$ (d) $\frac{\partial u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$

37) The solution of $(y-z)p + (z-x)q = x - y$ is ____________.

(a) $f(x+y+z) = xyz$ (b) $f(x^2+y^2+z^2) = xyz$

(c) $f(xyz) = x^2+y^2+z^2$ (d) $x+y+z=0$

38) Which of the following is a second order linear differential equation?

(a) $\left(\frac{dy}{dx}\right)^2 + xy = x$ (b) $x^2\frac{d^2y}{dx^2} + y = x^2$ (c) $\frac{d^2y}{dx^2} + y^2 = x^2$ (d) $\frac{d^2y}{dx^2} + y\frac{dy}{dx} = x$

39) The solution of differential equation $D^2y + 4y = 0$; $D \equiv \frac{d}{dx}$ is

(a) $y = e^{-2x} + e^{2ix}$ (b) $y = e^{4x} + e^{-4x}$ (c) $y = \cos 2x + \sin 2x$ (d) $y = \cos 4x - i\sin 4x$

40) If p is generalized momentum and q is generalized coordinate then $[p, q]$ = ________

(a) 1 (b) −2 (c) 2 (d) −1

Section – III [25]

1) Suppose f(z) is analytic for all finite values of z and $|f(z)| = c\,|z|^k$ as $|z| \to \infty$ for some $k \geq 1$ and for some $c > 0$. Then what is f(z)? Justify your answer.

2) Show that every measurable function is nearly equal to a continuous function.

3) Let V be a finite dimensional vector space over a field F and T : V → V be a linear transformation. Show that T is one to one iff T is onto.

4) Solve: $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = 0$.

5) Describe the method of solving Laplace's equation in two dimensions.

Section – IV [25]

1) Is it true that the Lebesgue integral of a measurable function generates a signed measure? Justify your answer.

2) Let G be a group with $o(G) = p^2$, where p is a prime number. Then prove that G must be abelian.

3) Prove that every metric space is normal.

4) Find the extremum of $I[y(x)] = \int_a^b y\sqrt{1+y'^2}\,dx$.

5) For the Lagrangian $L = \frac{I_1}{2}\left(\dot\theta^2 + \dot\phi^2\sin^2\theta\right) - mgl\sin\theta$; θ, ϕ generalized coordinates, find energy function. Is it conserved? Why?