

SARDAR PATEL UNIVERSITY

Ph. D. Entrance Test

2014

Friday, 8<sup>th</sup> August

Time Duration : 3 hours

Subject: Statistics

Note:

1) There are FOUR Sections, A. Objective type, B. Multiple Choice type, C. Short Answer type, and D. Long Answer type questions.

2) Attempt all the Sections. Sections A, B, C, and D carries 10, 40, 25, 25 marks respectively.

3) Write your answers in the space provided. No separate answer sheet will be given. Pages for rough work are given at the end of the booklet.

<b>A. Objectives</b>	
Answer all the ten questions. Each carries 1 Mark.	
1)	Let $X$ be a random variable with pdf $f(x, \theta) = q(\theta)d(x)$ if $0 < x < \theta$ and $f(x, \theta) = 0$ otherwise. Let $T$ be unbiased estimator for some function of $\theta$ . Does Cramer-Rao Lower Bound for $T$ exist? Justify.
2)	Let $X_1, \dots, X_n$ be a r.s. from $N(\mu, \sigma^2)$ , $\sigma^2$ is know. What type of test does exist? What is the form of the test?

3)	What is the idea behind the Neyman-Pearson theory of testing of hypothesis?
4)	Suppose a random sample of size $n$ is drawn from the population consists of $N$ units using SRSWR to estimate the population mean $\bar{Y} = \sum_{i=1}^N y_i/N$ . A variance of an unbiased estimator $\bar{y} = \sum_{i \in S} y_i/n$ is given by $Var(\bar{y}) = S_y^2/n$ . What is the standard error (s.e.) of this estimator?
5)	The characteristics roots of a variance covariance matrix are $\lambda_1 = 0.452$ , $\lambda_2 = 0.809$ , $\lambda_3 = 0.540$ , $\lambda_4 = 2.857$ , $\lambda_5 = .0343$ . What is the variance of the second principal component?
6)	Distinguish: central and non-central distributions

7)	Define a difference set for the construction of SBIBD (11, 5, 2).
8)	Why is the maximum rank( $C: l$ ), where $C$ is the $C$ -matrix of order $v$ and $l$ is a treatment contrast coefficient vector of size $v$ , equal to $v-1$ ?
9)	Mention the results of $C$ -matrix used in the statement of the necessary and sufficient conditions for connectedness, balancedness and orthogonality of a block design.
10)	Write down the contrast for ABC factorial effect in case of a $2^3$ factorial, as a function in AB factorial effect.

**B. Multiple Choice Questions**

Answer all the twenty questions.

Tick mark against the right answer and Justify. Each carries 2 Marks.

1)	Probability proportional to size (PPS) sampling is appropriate when (i) population is finite (ii) population units vary in sizes (iii) study variable and auxiliary variable are positively correlated (iv) all of the above
2)	Cluster Sampling and two-stage sampling are appropriate when (i) a list of elementary unit is not available, (ii) a list of groups of units is available, (iii) to increase precision of an estimator with fixed cost of surveys (iv) all of the above.
3)	The Lehmann-scheffe theorem is useful when an estimator is (i) sufficient statistic                      (ii) minimal sufficient statistics (iii) complete sufficient statistic      (iv) all of the above.

4)	A statistic T is called ancillary for $\theta$ or simply ancillary if it is (i) Sufficient statistic (ii) The statistic is complete (iii) The distribution of the statistic does not depend on $\theta$ (iv) Lower bound of the variance
5)	Whether a test is one sided or two sided depends on (i) simple hypothesis (ii) composite hypothesis (iii) null hypothesis (iv) alternative hypothesis
6)	A value of level of significance is set depending on the seriousness of (i) type I error (ii) type II error (iii) Power of test (iv) all of these
7)	For multi-parameter EFD a best test exists for two-sided alternative and is (i) MP test (ii) UMP test (iii) UMPU test (iv) none of the above
8)	To find the cut-off point in SPRT we must know (i) level of the test (ii) size of the test (iii) null distribution of $\lambda$ (iv) error probabilities
9)	A correlation coefficient between height and a joint effect of weight and age is known as (i) Karl-Pearson correlation coefficient (ii) Spearman's rank correlation coefficient (iii) Multiple correlation coefficient (iv) Partial correlation coefficient
10)	In context of two associate class PBIBD, the zero off-diagonal elements in the (2x2) P-matrix indicates that PBIBD has (i) treatments are first associates between group (ii) treatments are first associates within group (iii) treatments do not associate (iv) None of these
11)	Which one of these finite geometries results in SBIBD having $s^2+s+1$ treatment? (i) EG(2,s) (ii) PG (2,s) (iii) EG(s, s) (iv) PG (s, 2)
12)	The lack of fit F-test is given by the (i) Residual mean squares (ii) Pure error mean squares (iii) ratio of residual mean square to pure error mean squares (iv) ratio of pure error mean squares to residual mean squares

13)	A RBD is balanced because its variance of each elementary treatment contrast is (i) $\sigma^2$ (ii) $2\sigma^2$ (iii) $2\sigma^2/b$ (iv) $2\sigma$
14)	How many factors do a $2^{7-3}$ fractional factorial design has? (i) 7 (ii) 3 (iii) 8 (iv) 2
15)	The pair of a 2x2 factorial confounding AB and its variance is given by (i) $[a \ ab] [b \ 1], \sigma^2/4$ (ii) $[1 \ ab] [b \ a], \sigma^2/2$ (iii) $[1 \ ba] [a \ b], \sigma^2$ (iv) None of these
16)	The expected value of error sum of squares from a 'n' observation vector in fixed effect block design linear model is (i) $(n-v)\sigma^2$ (ii) $(n-v-b+g)\sigma^2$ , where r is the model parameters coefficient matrix (iii) $(n-v-b+1)\sigma$ (iv) $n\sigma^2$
17)	Given model $y_1 = \beta_1 + \beta_2 + e_1$ , $y_2 = \beta_2 + e_2$ , the two parameters are estimable (i) orthogonally (ii) independently (iii) with equal variance (iv) none of these
18)	The OLS linear regression is a generalized linear model with a link function (i) a probit link function (ii) an identity link function (iii) a logit link function (iv) None of these
19)	The least square estimate of $\sigma^2$ in linear model is (i) Error sum of squares (ii) Error mean squares (iii) Total variance of observations (iv) None of these
20)	The set of functions $y_1 - y_2 + y_3$ and $y_1 + y_2 + y_3$ carry what number of degrees of freedom. (i) 2 (ii) 1 (iii) 0 (iv) None of these

**C. Short Answer Questions (ANY 5)**

Each question carries 4 Marks.

1)	Write a note on the Horvitz-Thompson estimator for population total.
2)	Suppose the complete auxiliary information is available at estimation stage. Describe the situation and give appropriate estimator that is useful for estimation of the finite population mean.

3)	Verify whether a BIBD can contain a two associate PBIBD. Support your theoretical arguments by giving an example.
4)	Explain the difference between a factorial experiment and a split plot experiment. In general, which design is applied in the two experiments?

5)	Discuss on Need for the lower bound for variance with reference to Rao-Blackwell Theorem and Lehman Scheffe theorem.
6)	By stating various null and alternative hypotheses, class of distributions; mention for each of the null and alternative hypotheses which type of test exists and in which class of tests.



7)	How will you analyze multivariate data when the number of parameters exceeds the number of observations?
8)	By way of an example show that testing of hypothesis and construction of confidence interval are inter-related.

**D. Long Answer Questions (ANY 2)**

Each carries 12 ½ Marks.

- 1) Write critical note on the concept "Sufficiency". Also, discuss its roll in Statistical Inference.

2) Discuss benefits and limitations of each of the testing procedure: Neyman-Pearson Theory, Likelihood Ratio Test Principle, and Sequential Probability Ratio Test.

3) What are the various phases of sample surveys? Discuss each briefly. Also, discuss sampling and non-sampling errors associated with each phase.

4) Discuss the linear model theory used in the analysis of block designs.