SRI PADMAVATHI MAHILA VISVAVIDYALAYAM::TIRUPATI
(Women's University)

M.Phil./Ph.D. Entrance Test - September, 2012

DEPARTMENT OF APPLIED MATHEMATICS

Time: 3 Hrs **Max.Marks:100**

Answer any **Five** questions

Question No. 1 is compulsory

All questions carry equal marks **(5 × 20=100)**

1. (a) Define a Homomorphism between two groups. If G and H be two groups with identities e and e′ respectively and let $\phi : G \to H$ is a homomorphism then prove that (i) $\phi(e) = e'$ (ii) $\phi(x^{-1}) = (\phi(x))^{-1}$ for each $x \in G$

   (b) Define a metric space and give an example. Show that the union of arbitrary open sets is open

   (c) Define the continuity of a function in a metric space. Prove that the continuous image of a compact set is compact.

2. (a)Show that a finite integral domain is a ring.

   (b) Define a vector space. Prove that the set of all real valued continuous functions defined in the open interval (0, 1) is a vector space over the field of real numbers, with respect to the operations of addition and scalar multiplication defined as

   $(f + g)(x) = f(x) + g(x)$

   $(a f)(x) = a f(x)$, a is real with $0 < x < 1$

3. (a) Let X be a non-empty set. Show that the class of subsets of X consisting of empty set ϕ and all sets whose complements are countable, is a topology on X

   (b) Let X be second countable space. If a non empty open set G in X is represented as the union of a class $\{G_i\}$ of open sets then prove that G can be represented as a countable union of G_i's.

4. Use two phase simplex method to minimize $z = 6x + 21y$

subject to the constraints: $x + 2y \geq 3,$
$x + 4y \geq 4,$
$x \geq 0, y \geq 0.$

5. Solve the following transportation problem by Vogel's Approximation method

		To			Available
	5	8	3	6	30
From	4	5	7	4	50
	6	2	4	6	20
Demand	30	40	20	10	

6. (a) Define an analytic function. Show that f (z) = e^z is analytic everywhere in the complex plane and find f ′ (z).

(b) Show that the Mobius transformation w = 1/z is circle preserving

7. (a) State and prove Cauchy residue theorem

(b) Evaluate $\int_C \frac{z^2+4}{z-3}\,dz$ where C : $|z|$ = 5

8. (a) Use regula-falsi method to compute a real root of the equation $x^2 - 9x + 1 = 0$ if the root lies between 2 and 4

(b) Use fourth order Runge-Kutta method to solve the equation $10\frac{dy}{dx} = x^2 + y^2,$

y (0) = 1and find y in the interval 0 ≤ x ≤ 0.4 taking h = 0.1