RET/13/Test B

988	Mathematical Science

Question Booklet No. ....

(*To be filled up by the candidate by* ***blue/black ball-point pen***)

Roll No. | | | | | | | | | |

Roll No. (Write the digits in words) ....

Serial No. of OMR Answer Sheet ....

Day and Date ....

(Signature of Invigilator)

INSTRUCTIONS TO CANDIDATES

(Use only ***blue/black ball-point pen*** in the space above and on both sides of the ***Answer Sheet***)

1. Within 10 minutes of the issue of the Question Booklet, Please ensure that you have got the correct booklet and it contains all the pages in correct sequence and no page/question is missing. In case of faulty Question Booklet, bring it to the notice of the Superintendent/Invigilators immediately to obtain a fresh Question Booklet.
2. Do not bring any loose paper, written or blank, inside the Examination Hall *except the Admit Card without its envelope.*
3. *A separate Answer Sheet is given. It should not be folded or mutilated. A second Answer Sheet shall not be provided.*
4. Write your Roll Number and Serial Number of the Answer Sheet by pen in the space provided above.
5. *On the front page of the Answer Sheet, write by pen your Roll Number in the space provided at the top, and by darkening the circles at the bottom. Also, wherever applicable, write the Question Booklet Number and the Set Number in appropriate places.*
6. *No overwriting is allowed in the entries of Roll No., Question Booklet No. and Set No. (if any) on OMR sheet and Roll No. and OMR sheet No. on the Question Booklet.*
7. *Any changes in the aforesaid-entries is to be verified by the invigilator, otherwise it will be taken as unfair means.*
8. *This Booklet contains 40 multiple choice questions followed by 10 short answer questions. For each MCQ, you are to record the correct option on the Answer Sheet by darkening the appropriate circle in the corresponding row of the Answer Sheet, by pen as mentioned in the guidelines given on the first page of the Answer Sheet. For answering any five short Answer Questions use five Blank pages attached at the end of this Question Booklet.*
9. For each question, darken only one circle on the Answer Sheet. If you darken more than one circle or darken a circle partially, the answer will be treated as incorrect.
10. *Note that the answer once filled in ink cannot be changed.* If you *do not wish to attempt* a question, leave all the circles in the corresponding row blank (such question will be awarded zero marks).
11. For rough work, use the inner back page of the title cover and the blank page at the end of this Booklet.
12. Deposit *both OMR Answer Sheet and Question Booklet* at the end of the Test.
13. You are not permitted to leave the Examination Hall until the end of the Test.
14. If a candidate attempts to use any form of unfair means, he/she shall be liable to such punishment as the University may determine and impose on him/her.

Total No. of Printed Pages : 35

FOR ROUGH WORK

Research Entrance Test – 2013

No. of Questions : 50

Time : 2 Hours | ***Full Marks : 200***

Note : (1) This Question Booklet contains **40** Multiple Choice Questions followed by **10** Short Answer Questions.

(2) Attempt as many MCQs as you can. Each MCQ carries **3 (Three)** marks. **1 (One)** mark will be deducted for each incorrect answer. Zero mark will be awarded for each unattempted question. If more than **one** alternative answers of MCQs seem to be approximate to the correct answer, choose the closest one.

(3) Answer only **5** Short Answer Questions. Each question carries **16 (Sixteen)** marks and should be answered in **150-200** words. Blank **5 (Five)** pages attached at the end of this booklet shall only be used for the purpose. Answer each question on separate page, after writing Question No.

(4) ***For mathematical Science Students only :***

(i) This paper contains three Sections :

(A) Mathematical Section (Q. No. **11-40** & Short Answer Questions)

(B) Statistics Section (Q. No. **41-70** & Short Answer Questions)

(C) Computer Science Section (Q. No. **71-100** & Short Answer Questions)

A candidate has to attempt ***only one*** Section.

(ii) Q. No. **1 to 10** are ***compulsory*** to all.)

1. Most of the land precipitation and evaporation on earth takes place over the :

(1) land masses

(2) oceans and seas

(3) poles of the planet

(4) subtropical latitudes

2. The downstream portion of a river :

(1) generally becomes more sluggish

(2) usually has turbulent flows

(3) generally is of higher velocity, which is marked by reduced turbulence

(4) has lower discharges than do upstream portions

3. Which of the following is not a fatty acid ?

(1) Stearic acid (2) Palmitic acid

(3) Oleic acid (4) Phenyl acetic acid

4. Which of the following compounds is not an antibiotic ?

(1) Penicillin (2) Chloramine-T

(3) Streptomycin (4) Chloramphenicol

5. The acceleration with which a particle moves in a straight line, according to the law $v^2 = 4a(x\sin x + \cos x)$, v being the velocity of the particle at a distance x from a fixed point, is :

(1) 0 (2) $2\,ax\cos x$

(3) $4\,ax\cos x$ (4) $2\,ax\sin x$

6. If $\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} A \begin{bmatrix} 0 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then the matrix A is :

(1) $\begin{bmatrix} 3 & -4 \\ 3/4 & -1 \end{bmatrix}$

(2) $\begin{bmatrix} -13/4 & 3/2 \\ 5/4 & -1/2 \end{bmatrix}$

(3) $\begin{bmatrix} -17/4 & 3/4 \\ -7/4 & -1/4 \end{bmatrix}$

(4) $\begin{bmatrix} 5/4 & 11/4 \\ 3 & -9/4 \end{bmatrix}$

7. If the error in the measurement of radius of sphere is 0.3%, then the percentage error in the measurement of its volume is :

(1) 0.15%

(2) 0.6%

(3) 0.9%

(4) 0.03%

8. The resistance of series combination of two resistances is S. When they are joined in parallel, the total resistance is P. If S = nP, then the minimum possible value of n is :

(1) 3

(2) 4

(3) 2.1

(4) 0.89

9. Mitochondria are associated with the function of :

(1) cellular digestion

(2) circulation

(3) protein synthesis

(4) cellular respiration

10. In which parts of eyes, rods and cones are present ?

(1) Retina

(2) Iris

(3) Cornea

(4) Lens

(A) Mathematical Section

11. Let V be the space of all $n \times n$ matrices and $T : V \to V$ be a linear operator defined by $T(A) = \frac{A - A^T}{2}$; then nullity of T is :

(1) $2n$ (2) $\frac{n^2}{2}$ (3) $\frac{n(n+1)}{2}$ (4) $\frac{n(n-1)}{2}$

12. Let A be 3×3 matrix whose characteristic roots are 3, 2, –1. If B = $A^2 - A$, then $|B|$ is :

(1) 24 (2) –2 (3) 12 (4) –12

13. The set of all $x \in R$ for which the vectors $(1, x, 0)$, $(0, x^2, 1)$ and $(0, 1, x)$ are linearly independent in R^3 is :

(1) $\{x \in R : x = 0\}$ (2) $\{x \in R : x \neq 0\}$

(3) $\{x \in R : x \neq 1\}$ (4) $\{x \in R : x \neq -1\}$

14. How many elements of order 5 are in S_7 ?

(1) 120 (2) 21 (3) 504 (4) 24

15. The number of element in the field $\frac{z[i]}{(2+i)}$ is :

(1) 2 (2) 3 (3) 5 (4) ∞

16. The value of integral $\int_1^4 [\log_e x] dx$, where [ ] denotes greatest integer function, is :

(1) log 4 (2) 1/2 (3) $4 + e$ (4) $4 - e$

17. If f is defined by $f(x) = |x| + |x^2 - 1|$ $\forall x \in R$ then :

(1) f is discontinuous on R (2) f has local minima at $x = \pm 1/2$

(3) f has local maxima at $x = \pm 1/2$ (4) f has no local extrema

18. Residue of $f(z) = \frac{z^3}{(z-1)^4 (z-2)(z-3)}$ at $z = 3$ is :

(1) $\frac{101}{16}$ (2) –8 (3) 0 (4) $\frac{27}{16}$

19. The integral $\oint_{|z|=2} \frac{\cos z}{z^3} dz$ equals :

(1) πi (2) $-\pi i$ (3) $2\pi i$ (4) $-2\pi i$

20. The image of x = constant under the transformation $w = \sin z$ is :

(1) a parabola (2) a hyperbola

(3) a circle (4) an ellipse

21. $f(z) = e^y(\cos x + i \sin x)$ is :

(1) an entire function

(2) analytic in $x^2 + 4y^2 < 24$

(3) differentiable everywhere except $z = 0$

(4) nowhere analytic

22. Consider the following LPP : Max $z = x_1$ subject to $x_1 + x_2 \geq 1$, $x_1 - x_2 \geq 1$, $x_1 \geq 0$, $x_2 \geq 0$. The solution to its dual is :

(1) Unbounded (2) Infeasible

(3) Degenerate (4) Bounded

23. In an assignment problem with m-jobs and m-persons, the number of basic variables at zero level in a basic feasible solution is :

(1) m (2) m + 1

(3) m − 1 (4) 2m

24. The maximum number of saddle points for any particular λ in the given pay-off matrix is :

(1) 2

(2) 1

(3) 3

(4) 4

λ	6	−2
−1	λ	−7
−2	4	λ

25. The solution of the differential equation $x^2 \frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 6y = x$ is :

(1) $y = c_1x + c_2x^2 + \frac{x}{2}$

(2) $y = c_1x + c_2x^3 + \frac{x^2}{2}$

(3) $y = c_1x^2 + c_2x^3 + \frac{x^2}{2}$

(4) $y = c_1x^2 + c_2x^3 + \frac{x}{2}$

26. The initial value problem $(x^2 - x)\frac{dy}{dx} = (2x-1)y, y(x_0) = y_0$ has a unique solution if (x_0, y_0) equals :

(1) (2, 1) (2) (1, 1) (3) (0, 0) (4) (0, 1)

27. The general solution of the system of differential equations $\frac{dX}{dt} = MX + b$, where $X = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}; M = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $b = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ is given by :

(1) $e^{Mt}C + b$

(2) $e^{Mt}C + bt$

(3) $e^{Mt}C - b$

(4) $e^{Mt}C - bt$

(Where C is any 2×1 constant vector)

28. The complete integral of $x(1+y)p = y(1+x)q$ is :

(1) $Z = a(\log xy + x + y) + b$

(2) $Z = a(\log xy + x) + b$

(3) $Z = a(1 + x) + (1 + y)$

(4) $Z = ax + by + (a + b)xy$

29. A necessary condition for functional $I[y(x)] = \int_a^b f(x, y, y')dx$ to be an extremum is :

(1) $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$

(2) $\frac{\partial f}{\partial y} + \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$

(3) $\frac{\partial f}{\partial y'} - \frac{d}{dx}\left(\frac{\partial f}{\partial y}\right) = 0$

(4) $\frac{\partial f}{\partial y'} + \frac{d}{dx}\left(\frac{\partial f}{\partial y}\right) = 0$

30. Extremal of the functional $\int_{x_1}^{x_2} y\sqrt{1+y'^2}\,dx$ is attained on the :

(1) Catenary (2) Parabola (3) Circle (4) Sphere

31. Solution of the Poission's equation $u_{xx} + u_{yy} = -1$ in the square $|x| \le 1$, $|y| \le 1$ with $u = 0$ at $x = \pm 1$ and $y = \pm 1$ is :

(1) $u(x,y) = \frac{1}{16}(1-x^2)(1-y^2)$ (2) $u(x,y) = \frac{5}{16}(1-x^2)(1-y^2)$

(3) $u(x,y) = \frac{5}{16}(1+x^2)(1+y^2)$ (4) $u(x,y) = \frac{1}{16}(1+x^2)(1+y^2)$

32. The Newton divided difference polynomial which interpolate the data $f(0) = 1$, $f(1) = 3$, $f(3) = 55$ is :

(1) $8x^2 + 6x + 1$ (2) $8x^2 - 6x + 1$

(3) $8x^2 - 6x - 1$ (4) $8x^2 + 6x - 1$

33. Which one is true ?

(1) $\nabla = 1 - E^{-1}$ (2) $\nabla = 1 + E^{-1}$ (3) $\nabla = -1 + E^{-1}$ (4) $\nabla = -1 - E^{-1}$

where operators ∇, E have their usual meaning.

34. The order of the difference equation $u_{n+2} + u_{n+1} + 3u_n = 0$ is :

(1) 1 (2) 2 (3) 3 (4) does not exist

35. Solution of the integral equation $y(t) = t^2 + \int_0^t y(u)\sin(t-u)\,du$ is :

(1) $t^2 - \frac{t^4}{12}$ (2) $t^2 + \frac{t^4}{12}$ (3) $t - \frac{t^3}{6}$ (4) $t + \frac{t^3}{6}$

36. Fourier sine transform of e^{-x}, $x \ge 0$ is :

(1) $\frac{p}{1+p^2}$ (2) $-\frac{p}{1+p^2}$ (3) $\frac{1}{1+p^2}$ (4) $-\frac{1}{1+p^2}$

37. The number of independent components of Christoffel symbols are :

(1) $\frac{1}{2}n^2(n-1)$ (2) $\frac{1}{2}n^2(n+1)$ (3) $n^2(n-1)$ (4) $n^2(n+1)$

38. If E, F & G are components of first fundamental form of a surface, then the condition for the two directions given by equation $Pdu^2 + 2Qdudv + Rdv^2 = 0$ on the surface to be orthogonal is :

(1) $ER + 2FQ + GP = 0$ (2) $ER - 2FQ - GP = 0$

(3) $ER - 2FQ + GP = 0$ (4) $ER + 2FQ - GP = 0$

39. The attraction of a thin uniform rod AB upon an external point P is :

(1) $\frac{Gk\rho}{p}\sin\left(\frac{1}{2}APB\right)$ (2) $\frac{2Gk\rho}{p}\sin\left(\frac{1}{2}APB\right)$

(3) $\frac{Gk\rho}{p^2}\sin(APB)$ (4) $\frac{2Gk\rho}{p^2}\sin(APB)$

where G is gravitational constant, ρ & k be the density and cross section area of the rod respectively and p be the perpendicular distance from point P to rod AB.

40. Einstein field equation in empty space is :

(1) $R_{ij} - \frac{1}{2}Rg_{ij} = -\frac{8\pi G}{c^4}T_{ij}$ (2) $R_{ij} - \frac{1}{2}Rg_{ij} + \Lambda g_{ij} = -\frac{8\pi G}{c^4}T_{ij}$

(3) $R_{ij} = 0$ (4) $T_{ij} = 0$

where symbols have their usual meaning.

Attempt any five questions. Write answer in 150-200 words. Each question carries 16 marks. Answer each question on separate page, after writing Question Number.

1. Define cross ratio (z_1, z_2, z_3, z_4), where $z_i, i = 1,2,3,4$ are complex numbers. Show that the cross ratio (z_1, z_2, z_3, z_4) is real iff the four complex numbers z_1, z_2, z_3, z_4 lie on a circle or on a straight line.

2. Show that the equation $z^5 + 15z + 1 = 0$ has four roots in the annulus $\frac{3}{2} < |z| < 2$.

3. Find shortest path from the point A(–2, 3) to the point B(2, 3) located in the region $y \le x^2$, using methods of calculus of variation.

4. Show that a necessary and sufficient condition for a space curve to be spherical is that $\frac{\rho}{\sigma} + \frac{d}{ds}(\rho'\sigma) = 0$ at each point of the curve, where symbols have their usual meaning.

5. If A^i be contravariant component of a vector A, then show that $div\, A = \frac{1}{\sqrt{g}} \frac{\partial}{\partial x^i} (A^i \sqrt{g})$ where $g = \det(g_{ij})$.

6. Find out the equilibrium points and discuss the local stability of each equilibrium point for the following model :

$$\frac{dx}{dt} = rx\left(1 - \frac{x}{k}\right) - \alpha xy, x(0) = x_0 \geq 0.$$

$$\frac{dy}{dt} = e\alpha xy - \beta y, y(0) = y_0 \geq 0.$$

where $x(t)$ and $y(t)$ are the densities of prey and predator population respectively at time t. All parameters r, α, e, β & k are positive.

7. Show that Einstein field equation in empty space reduces to Laplace equation under certain approximations.

8. Solve the following differential equation using Laplace transform :

$$\frac{d^3y}{dt^3} - 3\frac{d^2y}{dt^2} + 3\frac{dy}{dt} - y = t^2e^t; \text{ where } y(0) = 1, \left(\frac{dy}{dt}\right)_{t=0} = 0, \left(\frac{d^2y}{dt^2}\right)_{t=0} = -2$$

9. Apply Runge-Kutta method to find an approximate value of y when $x = 0.2$, given that :

$$\frac{dy}{dx} = x + y, y = 1 \text{ when } x = 0.$$

10. Let $f : R^3 \to R^2$ satisfy the conditions $f(0) = (1,2)$ and $Df(0) = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \end{bmatrix}$. Let $g : R^2 \to R^2$ be defined by the equation $g(x, y) = (x + 2y + 1, 3xy)$. Find $D(gof)(0)$, where $(0) = (0, 0, 0)$.

(B) Statistics Section

41. A researcher has enough data available to estimate the probability density function $f_1(x)$ and $f_2(x)$ associated with populations π_1 and π_2 respectively. The cost of misclassification of an object (for which the measurement x can be recorded) in π_1 when it actually belongs to π_2, is 15 units whereas the cost of misclassification in π_2, when it actually belongs to π_1, is 5 units. It is also known that 20% of all the objects belong to π_2. An object having the corresponding observation x_0 is to be classified in π_2 if (R stands for $f_1(x_0)/f_2(x_0)$ in the lines given below) :

(1) $R \geq 1.33$ (2) $R \geq 0.75$

(3) $R \leq 1.33$ (4) $R \leq 0.75$

42. Two friends X and Y decide to meet for lunch at a pre-assigned place. Due to traffic conditions, they can only be sure of arriving between 12 : 00 noon and 1 : 00 p.m. They further decide to wait not more than 15 minutes from the time of their arrival or until the end of the hour. Let

S_1 : Probability that they will have their lunch together is 1/4

S_2 : Probability that Y arrived later than X given that they had their lunch together is 1/2.

Select the correct answer from the following codes :

(1) Both S_1 and S_2 are true

(2) S_1 is true but S_2 is false

(3) S_1 is false but S_2 is true

(4) Both S_1 and S_2 are false

43. A random sample $X_1, X_2, ..., X_n$ is drawn from a population having p.d.f.

$$f(x/\mu\theta) = \theta \exp[-\theta(x-\mu)], \text{ for } x > \mu, \theta > 0$$

$$= 0, \text{ otherwise}$$

Let S be min $(X_1, X_2, ..., X_n)$ and M be the sample mean. Then the MLE of μ and θ are respectively :

(1) S and M (2) S and M-S

(3) S and 1/M (4) S and 1/(M-S)

44. If X is a single observation from a Poisson distribution with parameter λ, consider the following in the context of estimation of $\exp(-3\lambda)$:

Assertion (A) : Unbiased estimators may be absurd.

Reason (R) : $(-3)^x$ is the only unbiased estimator of $\exp(-3\lambda)$.

Choose the answer from the following codes :

(1) Both A and R is correct and R is correct explanation of A

(2) Both A and R is correct and R is not correct explanation of A

(3) A is true but R is false

(4) A is false but R is true

45. Let $X_1, X_2, \ldots\ldots\ldots, X_n$ be a random sample from a normal distribution with mean θ and variance θ.

If $T_1 = \sum_{i=1}^{n} X_i$ and $T_2 = \sum_{i=1}^{n} X_i^2$, then

(1) T_1 and T_2 are jointly sufficient for θ

(2) only T_1 is sufficient for θ

(3) only T_2 is sufficient for θ

(4) neither T_1 nor T_2 is sufficient for θ

46. An experimenter needs six mice having a particular disease. He has a large collection of mice, out of which 30% are having that disease. He examines mice one by one until he gets six mice. The probability that the number of examinations required is more than 25 is :

(1) $\sum_{k=0}^{6} \binom{25}{k} (.3)^k (.7)^{25-k}$

(2) $\sum_{k=0}^{5} \binom{25}{k} (.3)^k (.7)^{25-k}$

(3) $\sum_{k=6}^{24} \binom{k-1}{5} (.3)^6 (.7)^{k-6}$

(4) $1 - \sum_{k=6}^{25} \binom{k-1}{5} (.3)^6 (.7)^{k-6}$

47. Independent random samples of 10 male and 10 female students of class IX were asked whether Mathematics should be made a compulsory subject at high school level. Eight males and six females were in favor of the proposal. We wish to test that the proportion (P_1) of male students favoring the proposal is more than the proportion (P_2) of female students favoring the proposal.

Assertion (A) : We can use Fisher-Irvin test for testing $H_0 : P_1 = P_2$ against $H_1 : P_1 > P_2$ by calculating the significance probability $P(X \geq 8 \mid X + Y = 14)$ under H_0.

Reason (R) : Even if $P_1 = P_2 = P$ (say) is not known, $P(X \geq 8 \mid X + Y = 14)$ under H_0, can be calculated using hyper-geometric distribution.

Choose the answer from the following codes :

(1) Both A and R are correct and R is a correct explanation of A

(2) Both A and R are correct and R is not a correct explanation of A

(3) A is true but R is false

(4) A is false but R is true

48. In the following rectangular game :

	B_1	B_2	B_3
A_1	10	7	10
A_2	2	4	5
A_3	4	p	6

it is known that the saddle point exists for the entry (A_1, B_2). The range of the unknown entry p will be :

(1) $p \leq 7$

(2) $4 \leq p \leq 10$

(3) $p > 7$

(4) $7 \leq p \leq 10$

49. In a state of India, the chance (in percent) of moving the population to a village, a town and a city is given in the following transition matrix :

To		Village	Town	City
From	Village	50	30	20
	Town	10	70	20
	City	10	40	50

Given that the present population has percentages 70, 20 and 10 in the village, town and city respectively, the percentage of population in village, town and city after two years will be :

(1) (25.2, 47.9, 26.9) (2) (43.2, 23.1, 34.4)

(3) (38.2, 39.7, 23.3) (4) (70.3, 20.5, 10.1)

50. In a systematic sample of size 10, taken from a population of size 100, if the 27th, 87th, 57th, 97th and 7th units of the population are included, then rest of the five units of the sample are :

(1) 17th, 67th, 37th, 77th and 47th units of the population

(2) 10th, 20th, 30th, 40th and 50th units of the population

(3) 1st, 2nd, 3rd, 4th and 5th units of the population

(4) Any five units of the population

51. If the coefficient of variations of the study variable Y and the auxiliary variable X in a population are 18 and 32 respectively, then for what range of the coefficient of correlation ρ between X and Y, the ratio method of estimation will be preferable over simple random sample ?

(1) $\rho < 0.63$ (2) $0.33 < \rho < 0.80$

(3) $\rho > 0.63$ but less than 0.85 (4) $\rho > 0.88$

52. Read the following statements carefully in context of the function given below :

$$F(x) = \begin{cases} 0, & \text{if } x < 0 \\ 3c^2, & \text{if } 0 \le x < 1 \\ 4c - 7c^2, & \text{if } 1 \le x < 2 \\ 9c-7c^2-1, & \text{if } 2 \le x < 3 \\ 1, & \text{if } 3 \le x \end{cases}$$

Assertion (A) : F(x) can be cumulative distribution function of a continuous positive random variable for properly chosen value of 'c'.

Reason (R) : For proper choice of 'c', F(x) is monotone and bounded between 0 and 1.

Select your answer from the following codes :

(1) Both A and R is true and R is correct explanation of A

(2) Both A and R is true but R is not correct explanation of A

(3) A is true but R is false

(4) A is false but R is true

53. The probability mass function of a random variable X is given below :

$f(x) = x/15; \quad x = 1, 2, 3, 4, 5$

$\quad = 0; \quad$ otherwise

Then the conditional probability that X lies between 1/2 and 5/2 given that X is greater than 1 is :

(1) 1/7 (2) 3/7

(3) 2/15 (4) 1/5

54. X is a random variable with E(X) = Var(X), then the distribution of X :

(1) is necessarily Poisson

(2) is necessarily Exponential

(3) is necessarily normal

(4) cannot be identified from the given information

55. A system of 5 identical units consists of two parts A and B which are connected in series. Part A has two units connected in parallel and part B has 3 units connected in parallel. All the five units function independently with probability of failure 0.5 then the reliability of the system is :

(1) 0.65625 (2) 0.34375 (3) 0.030125 (4) 0.96875

56. $\{X_n\}$ is a sequence of independently and identically distributed random variables with common variance σ^2. Let $Y_n = \frac{1}{n}\Sigma_{k=1}^{n} X_{2k-1}$ and $Z_n = \frac{1}{n}\Sigma_{k=1}^{n} X_{2k}$. Define $T_n = \sqrt{n}(Y_n - Z_n)$. Then $\{T_n\}$ is a sequence of random variables which is :

(1) independently and identically distributed

(2) independently but not identically distributed

(3) identically but not independently distributed

(4) neither independently and nor identically distributed

57. To examine whether two different skin creams have different effects on the human body, n randomly chosen persons were enrolled in a clinical trial Cream A was applied to one of the randomly chosen arms of each person, cream B to the other. What kind of design is this ?

(1) Completely Randomized Design

(2) Balanced Incomplete Block Design

(3) Randomized Block Design

(4) Latin Square Design

58. $\{X_n\}$ is a sequence of independently and identically distributed random variables with $P(X_i = 1) = p = 1 - P(X_i = 1), i = 1,2,........$ Let $Z = \frac{1}{500}\Sigma_{i=1}^{500} X_i$ and $\alpha = P(|Z - p| > 0.1)$. Then, for any value of p :

(1) $\alpha \geq .95$ (2) $\alpha \geq 0.1$ (3) $\alpha \leq 0.05$ (4) $.05 \leq \alpha \leq 0.95$

59. We have a data set consisting of 40 observations where an observation can be either 5 or 10. Which of the following statements are true ?

S_1 : The mean and median for the data will be same iff the variance for the data is zero.

S_2 : The mean and median for the data will always differ if the range for the data is 5.

Select the correct answer from the following codes :

(1) Both S_1 and S_2 are true (2) S_1 is true but S_2 is false

(3) S_1 is false but S_2 is true (4) Both S_1 and S_2 are false

60. X_1 and X_2 are the given independent observations drawn from a uniform population over $(0, \theta)$ and $(0, 1 + \theta)$ respectively. Then the sufficient statistics for θ is :

(1) $Min\{X_1, X_2\}$ (2) $Max\{X_1, X_2\}$

(3) $Min\{X_1, X_2 - 1\}$ (4) $Max\{X_1, X_2 - 1\}$

61. X is normally distributed with mean zero and variance σ^2 and Y independently follows exponential distribution with mean $2\sigma^2$. We wish to test $H_0 : \sigma^2 \leq 1$ against $H_1 : \sigma^2 > 1$ at α percent level of significance. The uniformly most powerful (UMP) test :

(1) Rejects H_0 when $X^2 + Y \leq \chi^2_{2;\alpha}$ (2) Rejects H_0 when $X^2 + Y \geq \chi^2_{2;\alpha}$

(3) Rejects H_0 when $X^2 + Y \leq \chi^2_{3;\alpha}$ (4) Rejects H_0 when $X^2 + Y \geq \chi^2_{3;\alpha}$

where $\chi^2_{n;\alpha}$ denotes the upper α percent point of chi-square distribution with n degrees of freedom.

62. A discrete random variable X has probability mass function $f(x \mid \theta)$. On the basis of a single observation, we wish to develop a test of size 0.05 for testing the hypothesis $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$. The value of the probability mass function $f(x \mid \theta)$ under $\theta = \theta_0$ and $\theta = \theta_1$ are given below :

	Value of the variable $X = x$			
	1	2	3	4
$f(x \mid \theta_0)$	.01	.04	.05	.90
$f(x \mid \theta_1)$	.80	.10	.05	.05

Then the most powerful test :

(1) Rejects H_0 if $x = 3$

(2) Cannot be obtained because there exists no test having size exactly equal to 0.05

(3) Has power more than 0.85

(4) Has power less than 0.85

63. The joint probability mass function of random variables X and Y

$$f(x,y) = \frac{\lambda^x e^{-\lambda} p^y (1-p)^{x-y}}{y!(x-y)!} \; y = 0,1,\ldots,x; \; x = 0,1,\ldots,\ldots$$

The marginal distribution of :

(1) X and Y both are Poisson

(2) X and Y both are binomial

(3) X is binomial and that of Y is Poisson

(4) X is Poisson and that of Y is binomial

64. The joint probability density function of (X, Y) is $f(x, y) = \exp\{-(x + y)\}$, for $0 < x < \infty$ and $0 < y < \infty$.

Statement S **:** X and Y are independently distributed.

Statement P **:** $P(X < Y \mid X < 2Y) = P(X < Y)$.

Choose your answer from the following codes :

(1) Both S and P are true

(2) S is true but P is false

(3) S is false but P is true

(4) Both S and P are false

65. Read the following in context of cumulative distribution function $F_{X,Y}(x,y)$:

$$F_{X,Y}(x,y) = 0, \text{ if } x + 2y \le 1$$
$$= 1, \text{ if } x + 2y > 1$$

Statement S : $F_{X,Y}(x,y)$ is cumulative distribution function of discrete random variables (X, Y).

Statement P : $P(X = 1/2 \cap Y = 1/4) = 1$.

Choose your answer from the following codes :

(1) Both S and P are true

(2) S is true but P is false

(3) S is false but P is true

(4) Both S and P are false

66. Random variables X_1, X_2 and X_3 are such that the correlation coefficients between X_1 and X_2, X_1 and X_3 as well as X_2 and X_3 are all equal to ρ. Then :

(1) ρ can take any value between -1 and $+1$

(2) ρ cannot be negative

(3) $\rho \geq -0.5$

(4) $\rho \leq -0.5$

67. In a survey of population consisting of N = nk units, a sample of n units is selected with a random start between 1 to k and then selecting every k^{th} unit. Then :

Assertion (A) : The variance of the unbiased estimate of the population mean cannot be estimated.

Reason (R) : No unbiased estimate of population mean exists.

Select your answer from the following codes :

(1) Both A and R are true and R is correct explanation of A

(2) Both A and R are true but R is not correct explanation of A

(3) A is true but R is false

(4) A is false but R is true

68. The joint probability density function of (X, Y) is $f(x, y) = \exp\{-(x + y)\}$, for $0 < x < \infty$ and $0 < y < \infty$, $P(1 < X + Y < 2)$ is equal to :

(1) $\int_0^1 \int_{1-x}^{2-x} f(x,y)dy\,dx + \int_1^2 \int_0^{2-x} f(x,y)dy\,dx$

(2) $\int_0^1 \int_{1-y}^{2-y} f(x,y)dx\,dy + \int_1^2 \int_0^{2-y} f(x,y)dx\,dy$

(3) neither (1) nor (2)

(4) Both (1) and (2)

69. In a Bayesian estimation problem for the mean λ of a Poisson distribution based on a sample of size n, the prior for λ is taken as Gamma having probability density function proportional to $\lambda^{\alpha-1}\exp(-\beta\lambda)$. Then :

(1) The prior is conjugate prior

(2) The prior is non-informative prior

(3) Posterior is also Poisson distribution

(4) The posterior mode is $n(n\bar{x} + \alpha)/(n + \beta)$

70. Which of the following statements is ***false*** ?

(1) Sophisticated statistical methods can always correct the results if the population you are sampling from is different from the population of interest, e.g. due to under-coverage.

(2) Nonresponse can cause bias in surveys because non-respondents often tend to behave differently from people who respond.

(3) Non-sampling errors are often bigger than the random sampling errors in surveys.

(4) Slight changes in the wording of questions can make a measurable difference to survey results.

Attempt any five questions. Write answer in 150-200 words. Each question carries 16 marks. Answer each question on separate page, after writing Question Number.

1. Let X be the lifetime, in hours, of a light bulb. The density of X is Gamma (2,1). At 9.00 a.m. the light bulb is installed in a room and is left on. At a random time during the lifetime of the light bulb, Chulbul Pandey enters the room.

   (a) What is the probability that Chulbul Pandey enters the room after 11.00 a.m. ?

   (b) What time do you expect Chulbul Pandey to enter the room ? (support with suitable calculations).

2. A monkey jumps to get a fruit from a tree. Probability that the monkey fails to get the fruit at nth attempt is $1/n$. Explain "How will you use Borel-Cantelli lemma to show that the monkey will eventually get the fruit with probability 1 ?"

3. Consider a sequence $\{X_n, n \geq 2\}$ of independent coin tossing trial with probability p for head (H) in a trial. Denote the states of X_n by 1, 2, 3 and 4 according as the trial number $(n - 1)$ and n results in HH, HT, TH and TT respectively. Show that $\{X_n, n \geq 2\}$ is a Markov Chain. Find P and show that P^m $(m \geq 2)$ is a matrix with all the four rows having same element.

4. What is principal component analysis ? Discuss its uses with example. Show that the principal components are all uncorrelated.

5. Show that the Horvitz-Thompson estimate is an unbiased estimate of the population mean. Obtain the variance of the estimate.

6. Define estimable parametric function. State the conditions of estimability. Show that such a function has unique linear estimator which is best unbiased.

7. Discuss how total and partial confounding can be done in 3^3- factorial experiment. Give the partition of degrees of freedom, content of the block and calculation of sum of squares in each case.

8. Explain the nature and application of Gompertz curve and modified Gompertz curve. Describe briefly one of the methods for fitting these curves to a given population data.

9. What do you mean by least favourable prior ? If there exists a prior $g(\theta)$ and d_g is a Bayes rule corresponding to $g(\theta)$ such that Bayes risk $R(g, d_g)$ is never less than the Sup $r(\theta, d_g)$. (sup is taken over whole parameter space and $r(\theta, d_g)$ denote the ordinary risk i. e. average loss, average taken over sample space,); then d_g is a minimax rule.

10. Let $X_{(1)}$, $X_{(2)}$ and $X_{(3)}$ are the order statistics of a random sample of size three from a uniform distribution over $(0, \theta)$, $0 < \theta < \infty$. Check for the bias of $4X_{(1)}$ and $2X_{(2)}$ as an estimator of θ. Which one has smaller MSE ?

(C) Computer Science Section

71. Which of the following is false about a probability density function $f(x)$ of a continuous random variable X ?

(1) $f(x) \geq 0$ (2) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

(3) $\int_{a}^{b} f(x)\,dx$ gives the probability of $a \leq X \leq b$

(4) $f(x)$ is probability value

72. A and B are two events such that $P(A/B) = P(B/A), P(A \cup B) = 1, P(A \cap B) > 0$. Which of the following is true about $P(A)$?

(1) $P(A) = \frac{1}{3}$ (2) $P(A) = \frac{1}{2}$

(3) $P(A) > \frac{1}{3}$ (4) $P(A) > \frac{1}{2}$

73. Cans of soft drinks cost \$0.30 in a certain vending machine. What is the expected value and variance of daily revenue (Y) from the machine, if the number cans sold per day (X) has expected value $E(X) = 125$ and variance Var (X) = 50 ?

(1) E (Y) = 37.5, Var (Y) = 50
(2) E (Y) = 37.5, Var (Y) = 4.5
(3) E (Y) = 37.5, Var (Y) = 15
(4) E (Y) = 125, Var (Y) = 15

74. Function $f(x)$ denotes the density function of random variable X. Which expression gives the moment (μ'_r) and central moment (μ_r) :

(1) $\mu'_r = \int_{-\infty}^{\infty} x^r f(x)dx, \mu_r = \int_{-\infty}^{\infty}(x-\mu'_1)^r f(x)dx; r = 1,2,3,....$

(2) $\mu'_r = \int_{-\infty}^{\infty} (x-x^r) f(x)dx, \mu_r = \int_{-\infty}^{\infty}(x^r-\mu'_1) f(x)dx; r = 1,2,3,....$

(3) $\mu'_r = \int_{-\infty}^{\infty} (x^r-x)^r f(x)dx, \mu_r = \int_{-\infty}^{\infty}(x^r-\mu'_1) f(x)dx; r = 1,2,3,....$

(4) $\mu'_r = \int_{-\infty}^{\infty} x^r f(x)dx, \mu_r = \int_{-\infty}^{\infty}(\mu'_1)^r f(x)dx; r = 1,2,3,....$

75. Which one is not basis of vector space $R^3(R)$?

(1) {(1, 0, 0), (0, 1, 0), (0, 0, 1)}
(2) {(1, 1, 1), (0, 1, 1), (1, 1, 0)}
(3) {(1, 1, 1), (1, 0, 1), (0, 1, 1)}
(4) {(1, 1, 1), (1, 0, 1), (3, 3, 3)}

76. If quadratic form $Q \equiv 2x_1^2 + 3x_2^2 + 6x_1x_2$ is expressed using matrix form $X'AX$, where $X' = [x_1, x_2]$, then matrix A is :

(1) $A = \begin{pmatrix} 2 & 6 \\ 0 & 3 \end{pmatrix}$
(2) $A = \begin{pmatrix} 2 & 0 \\ 6 & 3 \end{pmatrix}$
(3) $A = \begin{pmatrix} 2 & 3 \\ 3 & 3 \end{pmatrix}$
(4) All of the above

77. Which of the following statement is not true for square real symmetric matrix, A, if it is positive definite ?

(1) All eigen value of matrix A are positive definite

(2) All leading minors of matrix A are positive

(3) Main diagonal elements of matrix A are positive

(4) Leading minors of the matrix A may positive or negative

78. $\lim_{x \to 0} \frac{\sin(x)}{x}$ is :

(1) 1 (2) 0 (3) −1 (4) −1/2

79. If $\phi_1(x), \phi_2(x), ..., \phi_m(x)$ are solution of homogeneous linear differential equation $(D^n + b_1(x)D^{n-1} + ... + b_{n-1}(x)D + b_n(x))y = 0$, then among following which is also solution :

(1) $\phi_1(x)\phi_2(x)\phi_3(x)\phi_4(x)........, \phi_m(x); \ (m \le n)$

(2) $\phi_1(x)\phi_2(x) + \phi_2(x)\phi_3(x) + ...\phi_{n-1}(x)\phi_n(x)$

(3) $\phi_1(x)\phi_2(x) + \phi_2(x)\phi_3(x) + ...\phi_{m-1}(x)\phi_m(x); \ (m \le n)$

(4) $\phi_1(x) + 2\phi_2(x) + 3\phi_3(x) + ... + m\phi_m(x); \ (m \le n)$

80. Laplace transform of $f(x) = e^{ax}$ is :

(1) $\frac{1}{a-s}, (|s| > a)$

(2) $\frac{1}{s-a}, (\text{Im}(s) > a)$

(3) $\frac{1}{s-a}, (\text{Re}(s) > a)$

(4) $\frac{s}{s-a}, (|s| > a)$

81. Which expression gives the Z-transform, X(z), of the discrete sequence $\{x(n)\}_{n=-\infty}^{n=\infty}$?

(1) $X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n}$

(2) $X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{n}$

(3) $X(z) = \sum_{n=-\infty}^{\infty} z^{-n}$

(4) $X(z) = \frac{\sum_{n=-\infty}^{\infty} x(n)z^{-n}}{\sum_{n=-\infty}^{\infty} x(n)}$

82. A real sequence has Z-transform with one real pole located on the unit circle ($|z|=1$). The signal is :

(1) Oscillatory and decaying (2) Oscillatory and increasing

(3) Constant (4) Increasing

83. Gibbs phenomenon is related to function :

(1) Discontinuity (2) Periodicity

(3) Symmetry (4) Roundedness

84. If $a = 5$, $b = 10$ and $c = -6$ the value of expression $(a > b)$ && $(a < c)$:

(1) True (2) False

(3) True and False both (4) None of the above

85. Consider the following segment of C program :

```
int x, y, n;
x=1;
y=1;
if (n>0)
  x = x-1;
  y = y-1;
```

After execution of above program segment the value of x and y if $n=1$ is :

(1) $x=2, y=0$ (2) $x=1, y=0$ (3) $x=1, y=1$ (4) $x=2, y=1$

86. Consider the following segment of C program :

```
int x, y;
x=10;
y=7;
while(x%y>=0)
 {
   x = x+1;
   y = y+2;
 }
```

Number of times the body of while loop is executed :

(1) 2 (2) 3 (3) 4 (4) infinite

87. Consider the following segment of C program :

```
int i, j;
j=0;
for (i=0; i<=5; i=i+2/3)
{
  j = j+1;
```

The number of times the body of for loop is executed :

(1) 9 (2) 8 (3) infinite (4) 11

88. Consider the following segment of C program :

```
int a, b, c, d, e, f, g;
a = 15;
b=10;
c = ++a -b;
d = b++ +a;
e = a/b;
f=a%b;
a *=b;
```

Values of *a*, *b*, *c*, *d*, *e* and *f* after execution of above segments are :

(1) $a = 176, b = 11, c = 6, d = 26, e = 1, f = 5$

(2) $a = 170, b = 10, c = 5, d = 26, e = 1, f = 5$

(3) $a = 176, b = 11, c = 5, d = 26, e = 1, f = 5$

(4) None of the above

89. Negation of $P \rightarrow Q$ is :

(1) $\sim P \rightarrow \sim Q$ (2) $\sim Q \rightarrow \sim P$

(3) $\sim P \vee Q$ (4) $P \wedge \sim Q$

90. A and B are two logical statements. Statement B is logical implication of statement A if :

(1) $A \rightarrow B$ is tautology (2) $B \rightarrow A$ is tautology

(3) $A \rightarrow B$ is contradiction (4) $B \rightarrow A$ is contradiction

91. Let $A(x)$ is predicate. The logical expression $\sim(\forall x)\, A(x)$ is equivalent to :

(1) $(\exists x) \sim A(x)$ (2) $(\exists x)\, (\forall x) \sim A(x)$

(3) $(\exists x) \sim (\forall x)\, A(x)$ (4) $(\exists x)\, A(x)$

92. Let X, Y be set and $|X|$=m, $|Y|=n$, and $m<=n$. Total number of function from X to Y is :

(1) m^n (2) n^m

(3) $m*n$ (4) $m!*n!$

93. Given memory partition of 100K, 500K, 200K, 300K, and 600K (in order) and processes of 212K, 417K, 70K, and 96K (in order); using the first-fit partition algorithm, in which partition would the process requiring 96K be placed ?

(1) 500K (2) 200K

(3) 300K (4) 600K

94. If an integer needs two bytes of storage, then the maximum value of signed integer is :

(1) $2^{16}-1$ (2) $2^{15}-1$

(3) 2^{16} (4) 2^{15}

95. The postfix expression can be evaluated using a :

(1) stack (2) tree

(3) queue (4) linked list

96. If post order traversal of a binary tree is DEBFCA, then the pre-order traversal is :

(1) ABFCDE (2) ADBFEC

(3) ABDECF (4) None of the above

97. A binary search tree is binary tree :

(1) All items in the left subtree are less than root

(2) All items in the right subtree are greater than or equal to the root

(3) Each subtree is itself a binary search tree

(4) All of the above

98. Resources are allocated to the process on non-sharable basis is :

(1) Mutual Exclusion (2) Hold and wait

(3) No pre-emption (4) Circular wait

99. The word LL(I) is related to :

(1) Parsing method (2) Process scheduling

(3) Memory Management (4) Disk Management

100. Reliability of software directly dependent on :

(1) Quality of the design

(2) Number of error present

(3) Software Engineers Experience

(4) User Requirement

Attempt any five questions. Write answer in 150-200 words. Each question carries 16 marks. Answer each question on separate page, after writing Question Number.

1. Express AND logic using OR and NOT logic.

2. What are major phases of waterfall model of software development ? Which phase consumes maximum effort ?

3. Explain segmentation and paging scheme briefly.

4. Explain Newton-Rapson methods for finding roots of polynomial.

5. Write the following statements in symbolic form as well as write theri negation :

(i) If Jack fails high school, then he is uneducated.

(ii) If Jack reads a lot of books, then he is not uneducated.

6. Write C program using do – while and for loop to sum the following series for given value of n :

$$1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\dots\dots\dots+\frac{1}{n}$$

7. Find the mean and variance of a random variable if the corresponding density function is give by :

$$f_X(x)=\begin{cases}\dfrac{1}{b-a}, & a\le x\le b\\ 0\ , & \text{otherwise}\end{cases}$$

8. Consider the relation $R = \{(1, 2), (2, 1), (2, 3), (3, 2), (3, 4), (4, 3), (4, 2), (2, 4)\}$ on the set $X = \{1, 2, 3, 4\}$. Represent relation, R, using graph and give the adjacency matrix of the corresponding graph.

9. Find the Laplace transform of the function $f(x) = x^n$.

10. Evaluate the Z-transform of the following sequence :

$$x(n)=\left\{1,\frac{1}{2},\left(\frac{1}{2}\right)^2,\left(\frac{1}{2}\right)^3,\left(\frac{1}{2}\right)^4,\dots\dots\dots,\left(\frac{1}{2}\right)^n,\dots\dots\dots\right\}$$

Roll No. : ....

Q. No. :

Roll No. : ....

Q. No. :

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FOR ROUGH WORK