

Question Booklet No.

(To be filled up by the candidate by **blue/black ball-point pen**)Roll No.

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Roll No. (Write the digits in words)

Serial No. of OMR Answer Sheet

Day and Date

(Signature of Invigilator)**INSTRUCTIONS TO CANDIDATES**(Use only **blue/black ball-point pen** in the space above and on both sides of the **Answer Sheet**)

1. Within 10 minutes of the issue of the Question Booklet, Please ensure that you have got the correct booklet and it contains all the pages in correct sequence and no page/question is missing. In case of faulty Question Booklet, bring it to the notice of the Superintendent/Invigilators immediately to obtain a fresh Question Booklet.
2. Do not bring any loose paper, written or blank, inside the Examination Hall *except the Admit Card without its envelope*.
3. *A separate Answer Sheet is given. It should not be folded or mutilated. A second Answer Sheet shall not be provided.*
4. Write your Roll Number and Serial Number of the Answer Sheet by pen in the space provided above.
5. *On the front page of the Answer Sheet, write by pen your Roll Number in the space provided at the top, and by darkening the circles at the bottom. Also, wherever applicable, write the Question Booklet Number and the Set Number in appropriate places.*
6. *No overwriting is allowed in the entries of Roll No., Question Booklet No. and Set No. (if any) on OMR sheet and Roll No. and OMR sheet No. on the Question Booklet.*
7. *Any changes in the aforesaid-entries is to be verified by the invigilator, otherwise it will be taken as unfair means.*
8. *This Booklet contains 40 multiple choice questions followed by 10 short answer questions. For each MCQ, you are to record the correct option on the Answer Sheet by darkening the appropriate circle in the corresponding row of the Answer Sheet, by pen as mentioned in the guidelines given on the first page of the Answer Sheet. For answering any five short Answer Questions use five Blank pages attached at the end of this Question Booklet.*
9. For each question, darken only one circle on the Answer Sheet. If you darken more than one circle or darken a circle partially, the answer will be treated as incorrect.
10. *Note that the answer once filled in ink cannot be changed.* If you do not wish to attempt a question, leave all the circles in the corresponding row blank (such question will be awarded zero marks).
11. For rough work, use the inner back page of the title cover and the blank page at the end of this Booklet.
12. Deposit *both OMR Answer Sheet and Question Booklet* at the end of the Test.
13. You are not permitted to leave the Examination Hall until the end of the Test.
14. If a candidate attempts to use any form of unfair means, he/she shall be liable to such punishment as the University may determine and impose on him/her.

FOR ROUGH WORK

Research Entrance Test – 2013

No. of Questions : 50

Time : 2 Hours

Full Marks : 200

- Note :** (i) This Question Booklet contains **40** Multiple Choice Questions followed by **10** Short Answer Questions.
- (ii) Attempt as many MCQs as you can. Each MCQ carries **3 (Three)** marks. **1 (One)** mark will be deducted for each incorrect answer. Zero mark will be awarded for each unattempted question. If more than **one** alternative answers of MCQs seem to be approximate to the correct answer, choose the closest one.
- (iii) Answer only **5** Short Answer Questions. Each question carries **16 (Sixteen)** marks and should be answered in **150-200** words. Blank **5 (Five)** pages attached with this booklet shall only be used for the purpose. Answer each question on separate page, after writing Question No.

1. Most of the land precipitation and evaporation on earth takes place over the :
 - (1) land masses
 - (2) oceans and seas
 - (3) poles of the planet
 - (4) subtropical latitudes

2. The downstream portion of a river :
 - (1) generally becomes more sluggish
 - (2) usually has turbulent flows
 - (3) generally is of higher velocity, which is marked by reduced turbulence
 - (4) has lower discharges than do upstream portions

3. Which of the following is not a fatty acid ?
 - (1) Stearic acid
 - (2) Palmitic acid
 - (3) Oleic acid
 - (4) Phenyl acetic acid

4. Which of the following compounds is not an antibiotic ?
 - (1) Penicillin
 - (2) Chloramine-T
 - (3) Streptomycin
 - (4) Chloramphenicol

5. The acceleration with which a particle moves in a straight line, according to the law $v^2 = 4a(x \sin x + \cos x)$, v being the velocity of the particle at a distance x from a fixed point, is :
 - (1) 0
 - (2) $2ax \cos x$
 - (3) $4ax \cos x$
 - (4) $2ax \sin x$

6. If $\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} A \begin{bmatrix} 0 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then the matrix A is :

(1) $\begin{bmatrix} 3 & -4 \\ 3/4 & -1 \end{bmatrix}$

(2) $\begin{bmatrix} -13/4 & 3/2 \\ 5/4 & -1/2 \end{bmatrix}$

(3) $\begin{bmatrix} -17/4 & 3/4 \\ -7/4 & -1/4 \end{bmatrix}$

(4) $\begin{bmatrix} 5/4 & 11/4 \\ 3 & -9/4 \end{bmatrix}$

7. If the error in the measurement of radius of sphere is 0.3%, then the percentage error in the measurement of its volume is :

(1) 0.15%

(2) 0.6%

(3) 0.9%

(4) 0.03%

8. The resistance of series combination of two resistances is S. When they are joined in parallel, the total resistance is P. If $S = nP$, then the minimum possible value of n is :

(1) 3

(2) 4

(3) 2.1

(4) 0.89

9. Mitochondria are associated with the function of :

(1) cellular digestion

(2) circulation

(3) protein synthesis

(4) cellular respiration

10. In which parts of eyes, rods and cones are present ?

(1) Retina

(2) Iris

(3) Cornea

(4) Lens

11. The radius of convergence of the power series,

$$1 + \frac{a \cdot b}{1 \cdot c} z + \frac{a(a+1)b(b+1)}{1 \cdot 2c(c+1)} z^2 + \dots,$$

where a, b, c being complex numbers is :

- (1) 1 (2) 2 (3) 3 (4) 4

12. The function $f(z) = \sin\left\{2\left(z + \frac{1}{z}\right)\right\}$ has been expanded in the following series,

$$f(z) = a_0 + \sum_{n=1}^{\infty} a_n \left(z^n + \frac{1}{z^n}\right),$$

where $a_n, n \geq 1$, are given by :

(1) $a_n = \frac{1}{2\pi} \int_0^{2\pi} \sin(2 \cos \theta) \cos n\theta d\theta$

(2) $a_n = \frac{1}{2\pi} \int_0^{2\pi} \sin(4 \cos \theta) \cos n\theta d\theta$

(3) $a_n = \frac{1}{2\pi} \int_0^{2\pi} \sin(\cos \theta) \cos n\theta d\theta$

(4) $a_n = \frac{1}{2\pi} \int_0^{2\pi} \sin(4 \sin \theta) \cos n\theta d\theta$

13. Which of the following subsets of real numbers \mathbb{R} does not have the Cardinal number C ?

(1) $\{x \in \mathbb{R} : 0 < x < 1\}$

(2) $\{x \in \mathbb{R} : 0 \leq x < 2\}$

(3) $\{x \in \mathbb{R} : x \geq 0\}$

(4) $\{x : x \in \mathbb{N} = \text{Set of natural numbers}\}$

14. For a measure space (X, s, μ) , which of the following statements is not true ?

(1) If $\{A_n\}_{n=1}^{\infty}$ is a sequence of pairwise disjoint measurable subsets of X , then

$$\mu\left(\bigcup_{n=1}^{\infty} A_n\right) = \sum_{n=1}^{\infty} \mu(A_n)$$

(2) $\mu(\phi) = 0$ where ϕ is a null set

(3) If $A, B \in s$, $A \subseteq B$, then $\mu(A) \leq \mu(B)$

(4) If $\{B_n\}_{n=1}^{\infty}$ is a sequence of subsets of X , then

$$\mu\left(\bigcup_{n=1}^{\infty} B_n\right) = \sum_{n=1}^{\infty} \mu(B_n)$$

15. If $\sum_{n=0}^{\infty} a_n$ and $\sum_{n=0}^{\infty} b_n$ are each convergent series of real numbers then their

Cauchy's product series $\sum_{n=0}^{\infty} c_n$, where

$$c_n = \sum_{k=0}^n a_k b_{n-k}, \quad n \geq 0, \text{ is :}$$

(1) convergent but not absolutely convergent

(2) absolutely convergent

(3) not convergent

(4) divergent if $\sum_{n=0}^{\infty} a_n$ is absolutely convergent

16. Which of the following statements is not true ?

(1) If X is a non-empty set and $P(X)$ is a power set of X then $(X, P(X))$ is a topological space.

(2) If X is a non-empty set and $T = \{\phi, X\}$ then $\{X, T\}$ is a topological space.

(3) If X is an infinite set and $T = \{Y \subseteq X : Y \text{ is an empty set or } Y' \text{ is finite}\}$, then (X, T) is a topological space.

(4) If $X = \{a, b, c\}$ and $T = \{\phi, X, \{a, b\}, \{a, c\}\}$, then (X, T) is a topological space.

17. The number of elements of order 5 in the group $\mathbb{Z}_{25} \oplus \mathbb{Z}_5$ are :
- (1) 5 (2) 16 (3) 20 (4) 24
18. How many elements belong to the field $\mathbb{Z}[i]/\langle 2+i \rangle$, where $\mathbb{Z}[i]$ is the ring of Gaussian integers and $\langle 2+i \rangle$ is the ideal of $\mathbb{Z}[i]$ generated by $2+i$?
- (1) 2 (2) 5 (3) 7 (4) infinite
19. What is the smallest positive integer n such that there are four non-isomorphic abelian groups of order n ?
- (1) 24 (2) 36 (3) 48 (4) 64
20. Which of the following statement is correct ? In the ring of Gaussian integers $\mathbb{Z}[i]$:
- (1) 3 and 2 are irreducible elements, but 5 is not irreducible
(2) 3 and 5 are irreducible elements, but 2 is not irreducible
(3) 3 is an irreducible element but 2 and 5 are not irreducible
(4) 5 is an irreducible element but 3 and 2 are not irreducible
21. The degree of the splitting field of the polynomial $x^6 + 1$ over the field of rationals \mathbb{Q} is :
- (1) 3 (2) 8 (3) 6 (4) 4
22. If A is a nilpotent matrix, then A is similar to :
- (1) a diagonal matrix
(2) a scalar matrix
(3) a triangular matrix
(4) a triangular matrix whose entries on the main diagonal are all zero
23. If the motion of a fluid is irrotational, then the velocity vector \vec{q} satisfies the condition :
- (1) $\text{div } \vec{q} = 0$ (2) $\text{div } \vec{q} \neq 0$ (3) $\text{curl } \vec{q} = 0$ (4) $\text{curl } \vec{q} \neq 0$

24. The degree of freedom of a rigid body in motion with its two points fixed is :
 (1) 1 (2) 2 (3) 3 (4) 4
25. If at any point of a straight line, a system of forces acting at different points of a rigid body can be reduced to a single force and a single couple in which the axis of the couple coincides with the direction of the force, then the line is called :
 (1) Pitch (2) Central axis (3) Wrench (4) Screw
26. Among all the curves joining two given points $A(x_0, y_0)$ and $B(x_1, y_1)$, the one which generates the surface of minimum area when rotated about the x -axis is :
 (1) cycloid (2) parabola (3) hyperbola (4) catenary
27. If a rigid body is rotating about one of its own fixed point under no forces, then throughout the motion :
 (1) its kinetic energy remains constant but the angular momentum varies
 (2) both its kinetic energy and angular momentum vary
 (3) its kinetic energy varies but the angular momentum remains constant
 (4) both its kinetic energy and angular momentum remain constant
28. In the Hamiltonian formulation of a dynamical system, the equations of motion are in terms of :
 (1) one set of second order differential equations
 (2) one set of first order differential equations
 (3) two sets of second order differential equations
 (4) two sets of first order differential equations
29. The integral equation given by :

$$u(x) = f(x) + \int_a^b k(x, \xi) u(\xi) d\xi$$
 is known as :
 (1) Volterra integral equation of the first kind
 (2) Fredholm integral equation of the first kind
 (3) Volterra integral equation of the second kind
 (4) Fredholm integral equation of the second kind

35. Suppose that a curve $\alpha : [0, 4] \rightarrow \mathbb{R}^3$ is given by

$$\alpha(t) = (t - 3, \sqrt{t}, t\sqrt{t})$$

If $\phi : [0, 2] \rightarrow [0, 4]$ is given by

$$\phi(u) = u^2,$$

then the reparametrization β of the curve α by the map ϕ is :

- (1) $\beta(u) = (u^2 + 3, u^2, -3u^2)$ (2) $\beta(u) = (u^2 - 3, u, u^3)$
 (3) $\beta(u) = (u^2 - 3, u^2, u^3)$ (4) $\beta(u) = (u, u^2, u^3)$

36. Let $\alpha(s) = (r \cos ws, r \sin ws, hws)$ be a unit speed circular helix with

$$w = \frac{1}{\sqrt{r^2 + h^2}}.$$

then :

- (1) $k(s) = w^2 r, \quad \tau(s) = h^2 r$ (2) $k(s) = r^2 w, \quad \tau(s) = h^2 w$
 (3) $k(s) = w^2 r, \quad \tau(s) = w^2 h$ (4) $k(s) = h^2 r, \quad \tau(s) = h^2 w$

37. Which of the following statement is true for the cylinder ?

$$C^2 = \{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 = 1\}$$

and the sphere

$$S^2 = \{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 + z^2 = 1\} ?$$

- (1) C^2 and S^2 cannot be covered by a single surface patch
 (2) C^2 and S^2 can be covered by a single surface patch
 (3) C^2 cannot be covered by a single surface patch but S^2 can be covered by a single surface patch
 (4) C^2 can be covered by a single surface patch but S^2 cannot be covered by a single surface patch

38. The set of all straight lines in the Euclidean plane is a C^∞ manifold of dimension :

- (1) one (2) two (3) three (4) four

39. Consider the function

$$\alpha : \mathbb{R} \rightarrow \mathbb{R}^2$$

given by

$$\alpha(t) = (t^3 - 4t, t^2 - 4)$$

then :

- (1) α is an embedding with self intersection for $t = -2$ and $t = 2$, but α is not an immersion
 - (2) α is an immersion but it is not an embedding
 - (3) α is a submersion but not an immersion
 - (4) α is an embedding but not a submersion
40. Let $f: M_1 \rightarrow M_2$ and $g: M_2 \rightarrow M_3$ be C^∞ maps on C^∞ manifolds. Then the Jacobian maps of f and g :
- (1) are linear and satisfy the chain rule
 - (2) are linear but fail to satisfy the chain rule
 - (3) are non-linear and satisfy the chain rule
 - (4) are non-linear but fail to satisfy the chain rule

Attempt any five questions. Write answer in 150-200 words. Each question carries 16 marks. Answer each question on separate page, after writing Question Number.

1. Show that a subset of a topological space is closed if and only if it contains its boundary.
2. Prove that B, B' are Banach spaces over $k (= \mathbb{R} \text{ or } \mathbb{C})$ and T is a linear transformation of B into B' then T is continuous \iff its graph is closed.
3. Show that any field F of characteristic zero is a perfect field.
4. If N is a submodule of an R -module M such that N and M/N both are noetherian then show that M is noetherian.
5. Verify whether or not the transformation

$$P = \frac{1}{2}(p^2 + q^2), \quad Q = \tan^{-1}\left(\frac{q}{p}\right)$$

is a contact transformation ?

6. For an incompressible homogeneous fluid, the velocity distribution at the point (x, y, z) is given by

$$\begin{aligned}u &= -(c^2 y / r^2), \\v &= (c^2 x / r^2), \\w &= 0.\end{aligned}$$

Where r denotes the distance from the z -axis. Show that it is a possible motion and determine the surface which is orthogonal to stream-lines.

7. Reduce the initial value problem

$$\begin{aligned}\frac{d^2 y}{dx^2} &= \lambda y(x) + g(x) \\y(0) &= 1, y'(0) = 0\end{aligned}$$

into Volterra integral equation of the second kind.

8. Prove that the solutions of the Dirichlet problem depend continuously on the boundary data.
9. Consider the sphere

$$S^2(1) = \{(x^1, x^2, x^3) \in \mathbb{R}^3 \mid (x^1)^2 + (x^2)^2 + (x^3)^2 = 1\}$$

consider the collection

$$A = \{(\phi_N, U_N), (\phi_S, U_S)\},$$

where

$$U_N = S^2(1) - \{(0, 0, 1)\},$$

$$U_S = S^2(1) - \{(0, 0, -1)\}$$

and

$$\phi_N : U_N \rightarrow \mathbb{R}^2 \text{ and } \phi_S : U_S \rightarrow \mathbb{R}^2$$

are the stereographic projections. Then prove that A is an atlas for $S^2(1)$.

10. Let M be a Riemannian manifold with its Riemannian connection ∇ . Then prove that the Hessian Hf of a C^∞ function f on M is given by

$$(Hf)(X, Y) = \langle \nabla_X(\text{grad } f), Y \rangle, X, Y \in TM$$

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FOR ROUGH WORK

